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Dynamic Discrete Choice Modeling: Monte Carlo Analysis

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Abstract

Recent work on spatial models of commercial fishing has provided insights into how spatial regulatory policies (i.e. Marine Protected Areas) are likely to alter the fishing location choices of commercial fishermen and the efficiency of these policies. The applied studies have spanned a diverse range of fisheries, from sedentary to highly migratory species. This literature has largely ignored the inter-temporal aspects of commercial fishing site choice at the cruise level. Therefore, these models depict fishermen as if they are ignoring how a location choice on the first day of a cruise may have potentially important consequences for the rest of the cruise. For many fisheries, particularly highly migratory ones, fishermen might choose a dynamically optimal cruise trajectory rather than myopic day-by-day strategies. An econometric model that ignores the inter-temporal aspects of location choice will likely lead to erroneous conclusions regarding a vessel's response to spatial regulatory policies. A dynamic discrete choice model is developed herein that utilizes the same information conventionally used in static models but is entrenched in the principals of dynamic optimization (Bellman's principle). Using Monte Carlo analysis, we evaluate the relative performance of this estimator as compared to the conventional static model for a variety of conditions that mimic different fishery types.

JEL Codes: C15, C35, Q20, Q58

Keywords: Dynamic Discrete Site Choice, Monte Carlo Simulation, Commercial Fishing

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I. Introduction

Recently, fisheries management has undergone a paradigm shift as fisheries are no longer viewed as homogeneously distributed resources, but rather as heterogeneously distributed metapopulations (Sanchirico and Wilen 1999). This shift has facilitated the development of spatial management instruments for these resources. These instruments are either temporary or permanent and to date usually involve regional closures or regionally specified total allowable catches. Closures have risen to the forefront of the policy debate as marine protected areas (MPAs) have been utilized to protect biodiversity and potentially enhance fishery yields. With the advent of these well-defined spatial management regimes, it has become increasingly important to model the spatial behavior of fisherman in order to predict how they will respond to these policies. A majority of the models used to date employ a static discrete choice model to predict vessel dynamics within a fishing fleet. This paper proposes the use of an alternative discrete choice model that is entrenched in dynamic optimization and allows for vessel behavior to react not only to the state variables observed in a static random utility model, but to the expected future optimal spatial behavior given the current state variables.

Investigations into the dynamic behavior of individual fisherman began with the work of Bockstael and Opaluch (1983), when they modeled the supply response decision of fishermen within New England fisheries. From their study they discovered that New England fishermen responded positively to expected rents within a fishery, negatively to variability in expected returns, and that they possessed a sluggish response to the rent differentials between fisheries.[1] The multinomial logit framework they employed has become the benchmark by which other location choice studies have been either compared to or conducted with. It has been utilized to model location choice decisions in the pink shrimp fishery (Eales and Wilen 1986) and the New England groundfish fishery (Holland and Sutinen 1999, 2000), to name a few.[2] Extensions of the model have been made by endogenizing the participation decision to develop a nested-logit framework in order to investigate the welfare impacts of sea turtle conservation areas (Curtis and Hicks 2000), marine protected areas off of the California coast (Smith and Wilen 2003, 2004), and more recently closures along the east coast to protect essential fish habitat (Hicks et al. 2004). These applied studies have spanned a diverse range of fisheries and include species that are sedentary (Smith 2002; Smith and Wilen 2003, 2004) to highly migratory species (Curtis and Hicks 2000; Strand and Mastiaen 2000). For the most part, this literature has ignored the inter-temporal aspects of commercial fishing site choice.[3] That is, each location choice during a cruise is assumed to be made independently of others. Therefore, current state-of-the-art models typically assume fishermen ignore that a location choice on the first day of a cruise may have potentially important consequences for the rest of the cruise. For many fisheries, particularly ones with highly migratory species, it is likely that captains choose locations that are near other good locations. Further, it is likely that they choose a cruise trajectory rather than myopic day-by-day strategies. Consequently, we may observe a fisherman making a less than optimal location choice from the perspective of the myopic one period model that is rational when taken from the perspective of maximizing cruise profits. An econometric model that ignores these inter-temporal aspects of location choice, using static discrete choice models, will likely obtain biased parameter estimates and yield poor policy guidance.

The pitfall of utilizing static discrete choice models is ignoring the inter-trip temporal dynamics often present within many fisheries. To determine the optimal trip trajectory a vessel must solve the dynamic programming problem to determine not only their initial site selection but all future site selections assuming optimal future behavior given their current state. Solving this problem requires a substantial degree of forward looking behavior and myopic decisions may be inferior to the expected returns of a dynamically optimal action. Fishing trips will often involve many different hauls (i.e.- singular deployments of gear) and occur over a number of days. The number of hauls executed and the length of the fishing trip will depend on the fixed inputs of production employed (alternative vessel sizes and gear types), proximity to markets, parishability of the product, and biological characteristics of the resource. For instance, herring trips within the Northeast are predominately single day trips with a low number of individual hauls, whereas Tuna fishing trips in the Eastern Tropical Pacific Ocean are often well over a month and may contain a substantially larger number of hauls distributed over a very large area. Given these substantial differences in fishery types, it is necessary to develop spatial location decision models that are capable of predicting policy responses for either type of fishery. Because of the incredible diversity amongst the types of fisheries along the dimensions of trip length, variation of catch at fishing areas, and the travel distances involved in switching between areas, we strongly believe a Monte Carlo approach has advantages in allowing us to generalize as to when estimating the computationally more difficult dynamic model is preferred. We achieve this goal by simulating a number of different fishery conditions, in order to investigate when the dynamic model is most appropriate.

Previous researchers have developed a number of alternative estimation techniques to empirically model dynamic discrete choice models. These are classified as either full-solution (Rust 1987) or non-full solution methods (Keane and Wolpin 1994). The model developed herein is an extension of the full-solution model created by Rust (1987) when analyzing bus engine replacement decisions. This model has been utilized in the labor supply literature to estimate the impact of social security and medicare expenditures on retirement decisions (Rust and Phelan 1997). In addition, Rust's model has been applied to analyze the optimal stopping rule in forestry (Provencher 1995) and to a dynamic binary choice model of recreational fishing in the Great Lakes (Provencher and Bishop 1997).[4] The advantage of using a full-solution method to characterize the dynamic discrete choice problem is that it is structurally similar to the static multinomial logit model. In fact, the model developed herein nests the static discrete choice within the dynamic discrete choice model. When one assumes no forwarding looking behavior, or equivalently a discount factor of 0, the model degenerates to a static site selection model.

The difficulty of solving a dynamic discrete choice problem invariably depends on the time horizon over which the problem is to be solved, the number of alternative choices that may be made as well as the discount factor used to calculate the expected utility derived from all future optimal behavior. If the time horizon is small and/or the discount factor is extremely low the dynamic discrete choice model will converge to the solution of a static discrete choice model. To investigate this hypothesis, Monte Carlo analysis is conducted using alternative specifications of a simulated fishing fleet. Several key parameters are altered, the length of the fishing trip, the number of feasible sites a vessel can visit, the divergence between the true and assumed discount factor, and the relative importance of distance for the site choice problem.

In the following section we outline the model that can be used to econometrically estimate dynamic discrete choice decisions. Section three outlines the data generation process utilized within the Monte Carlo analysis and discusses the generalized results of the dynamic discrete choice model. Section four provides a summary of our results. Finally, the fifth section discusses the potential future applications of this model and the importance of its use in policy evaluation.

II. Dynamic Random Utility Modeling

Static choice modeling in fisheries has traditionally been conducted using the random utility framework developed by McFadden (1973). Given that the dynamic discrete choice model degenerates to the static model when the discount factor is 0, the static random utility model (RUM) will be developed first and then expanded to incorporate dynamic decision making. The static RUM model, as applied to commercial fishing, is prefaced by the assumption that agents are selecting among *N* alternatives in order to maximize current period rewards from fishing at time *t*. The current period rewards from choosing the j^{th} alternative in time period *t* can be expressed as,

$$\overline{R}_{jt} = R(\mathbf{x}_{jt}, \mathbf{z}_t; \beta) + \varepsilon_{jt} \tag{1}$$

where, β is a vector of preference parameters, $\mathbf{x}_{jt}$ is a vector of location specific characteristics and $\mathbf{z}_t$ is a vector of vessel specific characteristics that may change through time influencing the site selection decision.[5] The agent knows $\overline{R}_{jt}$ with certainty. The researcher is assumed to be able to observe a portion of the agent's current period rewards at each location ($R(x_{jt}, z_t; \beta)$), but is unable to observe the error term, ε_{jt}, which arises from latent information known by the agent but not observed by the researcher. Within the fisheries literature the location specific characteristics ($\mathbf{x}_{jt}$) often consist of the expected value of recent catches in an area (e.g. value of catch in an area in the past 30 days), seasonal expected values, expectations of the climatic conditions and the costs associated with moving to location *j* in time period *t* given a starting point. The vessel specific characteristics ($\mathbf{z}_t$) help control for heterogeneity in the production technology each vessel employs. Agents are assumed to select the alternative that maximizes their payoffs, thus selecting location *j* within time period *t* when

$$\overline{R}_{jt} \geq \overline{R}_{kt}, j \in N, \forall k \in N. \tag{2}$$

Therefore, the agent's unconditional payoffs at each time period *t* is

$$v_t = \max\{R(\mathbf{x}_{1t}, \mathbf{z}_t; \beta)) + \varepsilon_{1t}, R(\mathbf{x}_{2t}, \mathbf{z}_t; \beta)) + \varepsilon_{2t}, \ldots, R(\mathbf{x}_{Nt}, \mathbf{z}_t; \beta)) + \varepsilon_{Nt}\}. \tag{3}$$

If one assumes that error structure for each ε_{it} is independently and identically distributed with an extreme value distribution, the agent's choice decision can be econometrically estimated using the conditional logit model (McFadden 1973). The probability that choice *j* is selected in time period *t* can be expressed as,

$$\Pr(j) = \frac{\exp(R(x_{jt}, z_t; \beta))}{\sum_{k=1}^{N} \exp(R(x_{kt}, z_t; \beta))} \tag{4}$$

Recall, one of the arguments in the current period reward function in equation (1) for alternative *j* is the cost of moving to *k* given an initial starting point *i*. Consequently, the choice of any location at time *t* effectively locks in the travel costs that must be spent to reach each of the *N* sites at time *t+1*. Defining $x^{tc}_{i,j,t+1}$ as the cost of reaching *j* in period *t+1* given that *i* was chosen in period *t*, we can write the vector of travel costs the agent faces at time *t+1* as

$$x^{tc}_{t+1} = \left\{ x^{tc}_{i,1,t+1}, x^{tc}_{i,2,t+1}, \ldots, x^{tc}_{i,N,t+1} \right\}.$$

When travel costs are large relative to the value of catch and other site specific benefits, the importance of the current period choice on future payoffs is magnified via the cost of moving in the future.

For the dynamic model we derive below, the state space at time *t* is totally defined by location in that location links choices from period to period as described above. The state space could be expanded to include a number of items such as experience at sites, cumulative catch during the cruise, a state-dependent mechanism for updating expectations about payoffs at each site, or even temporal dependence of the error structure. In this paper, we explore a simple dynamic model where the state is totally defined by location because the model is computationally much less burdensome and because we believe it captures a majority of the dynamic considerations for most fisheries.

The dynamic random utility model (DRUM) we develop extends the static framework by allowing the agent to consider the impact of current choices on future payoffs while fishing. In the dynamic setting, the agent's objective function in time period *t* is to maximize

$$E\left[\sum_{\tau=t}^{T} \delta^{\tau-1} \sum_{k=1}^{N} \overline{R}_{k\tau} d_k(\tau) \,|\, S(\tau) \right] \tag{5}$$

where E[.] represents the agent's expectation operator, $\overline{R}_{kt}$ is the current period reward function defined in the static model, δ is the agent's discount factor, and $S(t)$ is the current state variables observable to the agent and the researcher in time period t and consists of all the factors that affect the agent's current period reward or the probability distribution of any future rewards.[6] To maximize this objective function, the agent selects a sequence of binary control variables, $d_k(t)$ for $t=0,\ldots,T$, indicating whether or not choice k is made in time period t ($d_k(t)=1$ if alternative k is chosen in period t).

Should the researcher assume no forward looking behavior, or equivalently, fully discounts the value of future payoffs on the cruise, the model degenerates to the static model summarized by equation (4). However, if forward looking behavior is incorporated into the model, the agent's maximum discounted reward function (hereafter referred to as the value function) can be expressed as

$$V(S(t),t) = \max_{\{d_k(t)\}_{k\in N}} E\left[\sum_{\tau=t}^{T}\delta^{\tau-1}\sum_{k=1}^{N}\overline{R}_k d_k(\tau) \mid S(t)\right] = \max_{k\in N}\{V_k(S(t),t)\} \tag{6}$$

where $V_k(S(t),t)$ represents the alternative specific reward function of choosing the k^{th} option in time period t. $V_k(S(t),t)$ depends on the state space observed by the agent in time period t and obeys the Bellman equation (Bellman 1957),

$$V_k(S(t),t) = \begin{Bmatrix} \overline{R}_k + \delta E[V(S(t+1),t+1) \mid S(t), d_k(t)=1] \ldots \text{for} \ldots t \le T-1 \\ \overline{R}_k \ldots \text{for} \ldots t = T \end{Bmatrix}. \tag{7}$$

which, given the definition of the observable components of the current period reward function can be rewritten as

$$V_k(S(t),t) = \begin{Bmatrix} R(\mathbf{x}_{kt}, \mathbf{z}_t; \beta) + \varepsilon_{kt} + \delta E[V(S(t+1),t+1) \mid S(t), d_k(t)=1] \ldots \text{for} \ldots t \le T-1 \\ R(\mathbf{x}_{kt}, \mathbf{z}_t; \beta) + \varepsilon_{kt} \ldots \text{for} \ldots t = T \end{Bmatrix}. \tag{8}$$

It is important to note that $V_k(S(t),t)$ includes the current period value from choosing area k and the value of all optimal future choices the agent makes given the current period choice. Therefore, the agent is actually selecting an optimal trajectory of discrete choices to maximize their expected discounted returns from a cruise of length *T*. In addition, in time period $t < T$ the agent knows $\overline{R}_{kt}$ but does not know with certainty $E\left[V(S(t+1),t+1) \mid S(t),d_k(t)=1\right]$ which captures the expected value of all future optimal behavior.

The addition of this third term, representing the value of all optimal future choices given the current period choice, presents two major hurdles that must be overcome to obtain an estimable model; 1) the third term must be solved using backwards recursion and 2) estimating the value function involves a multi-dimensional integral over the random elements $\{\boldsymbol{\varepsilon}\}$. Rust has shown that the later complication can be circumvented if one assumes that the errors are distributed multivariate extreme value, are conditional on the observable state variables and are serially independent. This is referred to by Rust as the conditional independence assumption and generates an additively separable value function which may be used to solve the backward recursion dynamics (Rust 1987).[7] Following Rust, the conditional value function can be expressed as follows

$$E[V(S(t+1),t+1) \mid S(t),d_k(t)=1] = \gamma + \ln\left[\sum_{k \in N} \exp(\overline{V}_k(\overline{S}(t+1),t+1)\right] \tag{9}$$

where γ is Euler's constant and

$$\overline{V}_k(\overline{S}(t+1),t+1) = R(\mathbf{x}_{kt+1},\mathbf{z}_{t+1};\beta) + \delta E\left[V(S(t+2),t+2) \mid S(t+1),d_k(t+1)=1\right]$$

represents the expected conditional value function in time period *t+1* for each of the *k* options given a realization of the state space. Invoking these assumptions allows us to write the alternative specific reward function for alternative *j* at time period *t* as

$$V_j(S(t),t) = R(\mathbf{x}_{jt},\mathbf{z}_t;\beta) + \varepsilon_{jt} + \delta\left[\gamma + \ln\left[\sum_{k \in N} \exp(\overline{V}_k(\overline{S}(t+1),t+1)\right]\right] \tag{10}$$

where $\boldsymbol{\beta}$ is the parameter vector and is analogous to the parameter vector estimated in the static model. The primary difference between the static and dynamic model is that the parameter vector $\boldsymbol{\beta}$ must fit the data not only for the current period but also for the additively separable expected value of all future time periods up to *T*. For instance, in time period *T-1* the parameter vector appears in two separate places, one within the value function and the other within the current period reward function.

$$V_j(S(T-1),T-1) = \max\left\{R(\mathbf{x}_{jT-1},\mathbf{z}_{T-1};\beta)+\varepsilon_{jT-1}+\delta\left[\gamma+\ln\left[\sum_{k\in S}\exp(R(\mathbf{x}_{kT},\mathbf{z}_{T};\beta))\right]\right]\right\} \tag{11}$$

As one proceeds further back in time towards the beginning of the cruise, the parameter vector and the discount factor become further nested in the value operator. Given the value function operator and the distributional assumptions on **ε**, the probability that an agent selects option *k* in time period *t* is represented as

$$\Pr(d_j(t)=1\,|\,\mathbf{x},\mathbf{z};\beta) = \frac{\exp\left(R(\mathbf{x}_{jt},\mathbf{z}_t;\beta)+\delta\left[\gamma+\ln\left[\sum_{k\in N}\exp(\overline{V}_k(\overline{S}(t+1),t+1)\right]\right]\right)}{\sum_{k\in N}\exp\left(R(\mathbf{x}_{kt},\mathbf{z}_t;\beta)+\delta\left[\gamma+\ln\left[\sum_{k\in N}\exp(\overline{V}_k(\overline{S}(t+1),t+1)\right]\right]\right)} \tag{12}$$

which can be estimated using a multinomial logit maximum likelihood function. In addition, it is important to note that the probability of selecting choice *j* in time period *t* degenerates to the static model given in equation (4) when the discount factor equals zero. Therefore, the static model is nested within the dynamic model when $\delta=0$. Estimation of this model is complicated by the dimensions of the choice set (*N*) in each period, the length of the time horizon *T* and the potential for state dependence in the state space *S(t)*. Within this paper we are primarily concerned with the spatial dynamics of location choice within a fishing trip. We investigate the performance of the DRUM model relative to the static model when the agent's true time horizon is less than the length of the fishing trip- the assumed time horizon in the DRUM model.

The endogenous evolution of the state space *S(t)* further complicates the analysis because at each time period either a stylized model of the evolution of the state space or an endogenously estimated Markovian transition probability matrix must be estimated given all feasible choices. Within the fisheries context the endogenous evolution of the state space would make considerable sense if one was modeling search behavior or the flow of information within the fishery over time

(Smith and Provencher 2003). However, the solution technique would be considerably more complicated because it would require either stochastic dynamic programming (Provencher 1995; Provencher and Bishop 1997; Smith and Provencher 2003) or the application of mutligrid algorithms (Rust 1997) to obtain estimates of the value function associated with optimal future behavior. It should be noted that a more complicate state space comes with a cost. All of these papers with the exception of Keane and Wolpin (1994) and Smith and Provencher (2003) are limited to analyzing a sequence of binary choices. Even these two papers are limited to a choice set containing only four alternatives or less and are forced to interpolate over the state space to minimize computational burden.

Given that our time horizon is considerably shorter than the inter-season or season-to-season search behavior that may exist within fisheries, we assume that for each haul the individual updates their information about areas, *S(t)*, given current activity by the fleet. However, when calculating the value function for future periods, information is treated as fixed. Although this may appear to be a relatively restrictive assumption, the information incorporated is identical to that conventionally utilized in the static discrete choice models currently used in most spatial discrete choice models within the fisheries literature. Therefore, the dynamic discrete choice model developed represents an alternative estimation technique given the same information that would be utilized in static models. Also any marginal improvements in the predictive accuracy of the dynamic discrete choice model can be attributed to the forward looking behavior of the vessels and not some alternative specification of the information processed by vessels. Presumably the implications of this assumption can be empirically investigated, but we leave this for future research.

Welfare Measurement and Dynamic Discrete Choice Models

Since this work is largely motivated by spatial management that curtails fishing activity in specific areas of the ocean, we compare the relative performance of welfare measures from the static and dynamic models for closures of fishing sites. Consider the closure of one area of the ocean in the context of the static model and an agent starting from a port ready to embark on a cruise. The static model assumes that the agent fails to consider how current choices may impact the overall returns over the course of the entire cruise. Therefore, in the static model the agent would need compensation for each location decision made during the cruise. An artifact of using the static model for welfare measures when a series of choices are linked by travel costs is that closing areas are likely to force agents to reoptimize their best spatial choice. It is likely that fishermen will be observed fishing in closed areas if data are collected prior to the spatial closure regulation. Following a closure, a fisherman would not choose these areas; consequently, the researcher might calculate the static welfare measure assuming that fishermen optimally avoid these areas.

Consider a spatial closure policy that reduces the set of feasible fishing sites from N^0 to N^1. Following the estimation of the static model and the recovery of the parameter vector from the static model, denoted by $\hat{\beta}_S$, the compensating variation (CV) that would equate the expected returns in period t can be written as,

$$\mathrm{CV}(\mathbf{x}_t, \mathbf{z}_t, t) = \frac{\ln\left[\sum_{k \in N^1} e^{R(\mathbf{x}_{kt}, \mathbf{z}_t; \hat{\beta}_S)}\right] - \ln\left[\sum_{k \in N^0} e^{R(\mathbf{x}_{kt}, \mathbf{z}_t; \hat{\beta}_S)}\right]}{-\hat{\beta}_S^{REV}} \tag{13}$$

where $\hat{\beta}_S^{REV}$ is the estimated marginal utility of income, estimated over the elements of $\mathbf{x}_{kt}$ that influence the amount of dollars the agent will receive at alternative k in time t.[8] Recall that in period 1, the starting point is always port. For periods $1 < t \leq T$, the choice the individual would have made with closures can't be observed. Consequently, for each period in the cruise next period's vector of travel costs can't be assigned with certainty, since choices may have been altered following a closure. To enable a complete comparison with our proposed dynamic welfare measure, we propose two static measures for CV:

<u>Measure 1:</u> This measure assumes that spatial behavior is not reoptimized following the closure. The data describing the baseline situation, the matrix **x**, is defined over observed locations and travel costs for next periods' choices are calculated accordingly. Using equation (13), define the compensation over the entire cruise that equates static payoffs at each period:

$$\mathrm{Total\_CV}^1 = \sum_{\tau=1}^{T} \delta^{\tau-1} \mathrm{CV}(\mathbf{x}_\tau, \mathbf{z}_\tau, t) \tag{14}$$

<u>Measure 2:</u> Assume that agents do reoptimize their location given the new choice set, S^1 and the static model parameter estimates. Denote $\mathbf{x}^2$ as the baseline matrix of alternative specific data

given that the agent is allowed to reoptimize their location choice given the closure. Using the static choice model probabilities, we predict a cruise trajectory and track travel costs during the predicted cruise according to the new optimal behavior. Using equation (13), define the compensation over the entire cruise that equates static payoffs at each period:

$$\text{Total\_CV}^2 = \sum_{\tau=1}^{T} \delta^{\tau-1} CV(\mathbf{x}_\tau^2, \mathbf{z}_\tau, t) \tag{15}$$

Since the latter measure eliminates closed areas and allows the agent to choose optimally (given the static framework) over the remaining open areas, $\text{Total\_CV}^1 \geq \text{Total\_CV}^2$.[9] Notice that both measures discount compensation during the cruise at the same rate as that assumed in the dynamic model.

The difficulties with the static welfare analysis are apparent since choice occasions are awkwardly linked and the welfare measures are heavily dependent on what one assumes agents might do given area closures. The dynamic model links choice occasions explicitly and allows the analyst to measure welfare changes in a consistent way because agents are modeled as reacting optimally for the duration of a cruise following a policy change. However, this means that the calculation of the welfare measure necessarily requires the calculation of the value function for each agent and policy under consideration. The dynamic nature of the model allows the measurement of a payment made in time period *t* that equates the value of the remainder of a cruise with and without the area closure. From equation (6) we can define a payment CV_t^D as the money amount that satisfies

$$V(S(t), t) \mid N^0 = V(S(t), CV_t^D, t) \mid N^1$$

Given our assumptions this has the closed form solution,

$$CV_t^D = \frac{\ln\left(\sum_{k\in N^1}\exp\left(R(\mathbf{x}_{kt},\mathbf{z}_t;\hat{\beta}_D)+\delta\left[\gamma+\ln\left[\sum_{k\in N^1}\exp(\overline{V}_k(\overline{S}(t+1),t+1)\right]\right]\right)\right)- \ln\left(\sum_{k\in N^0}\exp\left(R(\mathbf{x}_{kt},\mathbf{z}_t;\hat{\beta}_D)+\delta\left[\gamma+\ln\left[\sum_{k\in N^0}\exp(\overline{V}_k(\overline{S}(t+1),t+1)\right]\right]\right)\right)}{-\hat{\beta}_D^{REV}} \tag{16}$$

where, $\hat{\beta}_D$ is the vector of estimated parameters from the dynamic model and $\hat{\beta}_D^{REV}$ is the estimated marginal utility of income. Using this framework, it is possible to calculate the payment required to equate the value of the entire cruise both with and without the area closure by evaluating (16) when *t=1*.

Estimating the Dynamic RUM Model

Estimation of the likelihood function is a nested procedure that synthesizes the estimation of the value function for each decision period given the current vector of parameter estimates, $\widehat{\boldsymbol{\beta}}$, with a gradient based search algorithm for the parameter vector that maximizes the likelihood function.[10] A stylized depiction of the solution algorithm is as follows.[11]

(1) Starting values for the parameter estimates are established and used in the calculation of the value function.[12]
(2) The last choice period, *T*, within the trip is determined and the value function is recursively estimated following Bellman's principal and the additively separable value calculations proposed by Rust (1987), up to the current haul at time *t*. This calculation is performed for each vessel and haul in the sample.
(3) The choice probabilities are re-estimated using the updated value function calculations and maximum likelihood is used to obtain new parameter estimates.
(4) Following the estimation of new parameter estimates, all steps are repeated until the parameter estimates obtained converge to those that maximize the likelihood function using a gradient based optimization algorithm.

Ideally one would like to be able to endogenously estimate the discount factor within the solution algorithm outlined above. However, as pointed out by Rust (1987) the discount factor is often highly collinear with the parameter vector estimates, therefore making it extremely difficult to precisely estimate the discount factor. This high degree of collinearity results from the nesting of the parameter vector within the value function calculations and can be seen in equation (11). A nearly identical change in the probability of choice *k* being selected could be achieved by increasing(decreasing) the magnitude of the discount factor or decreasing(increasing) the magnitude of the parameter vector. Therefore, it is often necessary to assume a discount factor to use within the model and then conduct a sensitivity analysis on the choice of the discount factor selected. Alternatively, one could conduct a grid search over the discount rate from 0 to 1 using a pre-specified interval and then compare the log-likelihoods to determine the model which obtains the best fit.[13] The advantage of this procedure is that it will allow the researcher to obtain the static model estimates, $\delta=0$, and the DRUM estimates for comparison.

Within the Monte Carlo analysis we are able to generate a data set with a known discount factor and then investigate how deviations away from the true discount rate affect the results. The following section discusses the data generation process utilized to investigate the performance of the dynamic discrete choice model proposed within this paper. In addition, the Monte Carlo results are discussed to determine the relative performance of the static and dynamic models.

III. Monte Carlo Analysis

Within this research inquiry we are primarily interested in investigating the relative performance of the dynamic versus the static random utility model when we believe that vessel behavior may be dynamic. There are seven primary research questions we address in our Monte Carlo analysis.

Conjecture One: As the duration of the cruise increases the dynamic random utility model is better suited to estimating the inter-cruise dynamic behavior than the static model.

The average length of a cruise will vary across different fisheries and may extend anywhere from a single day trip to over a month. As the length of the trip increases, the dynamic behavior of a vessel becomes more important as site selection will invariably be linked with the feasible fishing grounds they may enter in the following period. In addition, should a site have high expected net returns and yet be exceptionally far away from a vessel's current location, the trajectory they select to enter these fishing grounds becomes exceptionally important. On the other end of the spectrum, a trip having only one haul does not possess any forward looking behavior because the value function associated with optimal future behavior is zero. The static and dynamic models will therefore yield identical estimates of the parameter vector. To investigate conjecture one, we use three alternative cruise lengths in the analysis: 10, 20, and 40 haul trips.

Conjecture Two: As the number of potential fishing sites increases, the dynamic random utility model is better suited to estimating the inter-cruise dynamic behavior than the static model.

The degree of forward looking behavior will invariably be linked to the number of alternatives that a vessel has at each point in time. For instance, if a vessel has only one location to fish in for each period the degree of forward looking behavior in their cruise trajectory would be indistinguishable from myopic behavior. However, as the number of choice alternatives increases, as well as the distances between these alternatives, the forward looking dynamics becomes increasingly more important. To investigate this conjecture we look at three alternative site selection models: 5, 10, and 20 potential sites.

Conjecture Three: Deviations in the discount factor assumed by the researcher, and implemented in the DRUM, from the true discount factor compromise the performance of the dynamic random utility model.

The discount factor defines the weight that fishermen assign to the value of their expected future optimal behavior and therefore their degree of forward looking behavior. Should the researcher assume that a vessel's discount factor is substantially greater than the true discount factor, the intertemporal decision dynamics will be straight jacketed by the dynamic model. The extreme case of this would arise if the true discount factor was zero- myopic behavior- and the researcher assumes a high discount rate. This will not only yield a poor fit to the true underlying behavior but it will cause erroneous dynamic effort predictions. Should this occur any policy recommendations made under the mis-specified discount factor will be inappropriate. To investigate this conjecture we estimate the dynamic random utility model utilizing small deviations from the true discount factor (± 0.5) and evaluate the performance of the estimator compared to when the assumed discount rate equals the truth.

Conjecture Four: As the importance of distance between sites increases, the performance of the dynamic model will increase relative to the static model.

Current location effectively defines the vector of travel costs next period. As the relative importance of these costs increase, the better the dynamic estimator will perform compared to the static model. This occurs because the dynamic model will increase the probability of choosing sites that are situated near other valuable sites, whereas the static model looks at each site's payoffs without regard to what is happening in close proximity to a candidate site.

Conjecture Five: Welfare estimates generated from a RUM model exceed those from the DRUM model.

Because the DRUM model explicitly allows for vessels to be forward looking in their selection of an optimal cruise trajectory, they will fully incorporate the closure into their behavior. Therefore, the closed area will alter the payoffs for all future potential sites. The vessel will optimally react and select a dynamic trajectory that reduces their exposure to economic losses. The static model, on the other hand does not allow the closure to effect future behavior and therefore the myopic behavior will increase the opportunity costs of a removed fishing site. To investigate this conjecture, we estimate the CV for the static and dynamic models varying the degree of heterogeneity in the site characteristics and the magnitude of the distance scale. In doing so, we can investigate which fishery characteristics yield a more substantial divergence in CV estimates.

Conjecture Six: Asymmetry in the forward looking behavior assumed by the researcher and possessed by the fishermen compromise the performance of the DRUM model as well welfare estimates.

Presumably in many fisheries a vessel's forward looking behavior may be less than the length of the cruise. This would imply that in every decision period the Bellman equation expressed in equation (7) would be capped at the time horizon possessed by the fisherman. Given that this information on the true time horizon is most likely not observed by the researcher, asymmetry in the assumption of forward looking behavior and the truth will presumably compromise the researcher's ability to competently estimate the CV as well as the true spatial behavior. To investigate this we generate data sets within the welfare section of the Monte Carlo analysis that assume fishermen possess a forward looking time horizon of 1, 3, 5, 10 and 20 periods while the researcher assumes it is 20 periods. This will allow us to investigate the impact of this asymmetry not only on the DRUM's ability to predict spatial behavior and welfare losses from closures.

Conjecture Seven: The less forward looking fishermen are the better the static model approximates their spatial behavior and the welfare losses.

The performance of the DRUM estimator is driven by the degree of forward looking behavior possessed by the fishermen. At the extreme, if fishermen are not forward looking the static random utility model should provide the best estimates. Therefore, one would expect the performance of the static model to be better than that of the DRUM estimator when the true time horizon is 1 and the researcher assumes it is greater than 1.

Given that the DRUM utilizes the same information as the static random utility model, testing the performance of the estimator via Monte Carlo analysis is extremely tractable. The data generation process utilized initializes the current period reward function for each location (either 5, 10 or 20) across the length of the cruise (either 10, 20 or 40 hauls), $R(\mathbf{x}_{kt}, \mathbf{z}_t; \boldsymbol{\beta})$ as follows,

$$R(\mathbf{x}_{kt}, \mathbf{z}_t; \boldsymbol{\beta}) = \beta_0 x_{0kt} + \beta_1 x_{1kt} + \beta_2 x_{2kt} \tag{17}$$

where x_{okt} is the Euclidean distance a vessel travels to location k given their current location and the observation matrices, x_{1kt} and x_{2kt} , are distributed $N(\mu_k,\sigma_k^2)$ random variables and the parameter vector is initialized at *[-0.5, 0.2, 0.5]*. The return functions specified in equation (17) are used to obtain the value functions, $\overline{V}_k(\overline{S}(t+1), t+1)$, in order to construct $V_k(S(t), t)$ expressed in equation 7. The method used to construct equation 7 is similar to the solution algorithm utilized within the dynamic random utility estimator. The final haul is initialized, either 10, 20 or 40, and then backwards recursion is used to construct all the T value functions used in the estimator across all k locations. In addition, a discount factor, δ, of 0.9 was used to construct the value functions. Finally, by drawing a random variate from a generalized extreme value distribution, the following error term was added to $V_k(S(t), t)$ to construct equation (8) in order to generate choices consistent with the maintained assumptions of the model,

$$\varepsilon_{kt} = -\left(\frac{1}{\lambda}\right)\ln\left(\ln\left(\frac{1}{u_{kt}}\right)\right) \tag{18}$$

where u_{kt} is a $U(0,1)$ random variable and λ is the scale parameter that takes a value of 1 within the Monte Carlo simulations.[14] Figures 1, 2, and 3 depict the spatial layout of potential fishing locations for the 5, 10, and 20 location models respectively. We have constructed fishing grounds that are relatively symmetric and where the distance from one site to an adjacent site is roughly equal. Our constructed fishing grounds are similar to those commonly defined in empirical studies of fishing location choices. Each location's coordinates were used to calculate the Euclidean distance to all other potential fishing locations. Tables 1 and 2 illustrate the distributional assumptions of the variables in the model, x_{1kt} and x_{2kt}, when the distance scale is set equal to 1 (compact fishery) and 10 (large fishery) respectively. For each Monte Carlo conducted we generated a total of 4000 observations by varying the length of cruise and the number of cruises simulated so that their product equals 4000 observations. In addition, for each parameterization we conducted 500 Monte Carlo simulations.

To investigate the welfare impacts of a closure we simplify the Monte Carlo analysis by truncating the observation matrix to contain only x_{0kt} and x_{2kt} and retain their corresponding β coefficients, $[-0.5, 0.5]$, and focus solely on the 5 location model. We also assume two different types of fisheries, a homogeneous and a heterogeneous fishery. The homogeneous fishery possesses identical expected catches across each location, whereas in the heterogeneous case two regions possess a higher expected catch and also a lower standard deviation, locations 1 and 5 (See Table 9). To investigate conjectures 6 and 7 we generate data sets which possess a true time horizon of 1, 3, 5, 10 and 20 periods of forward looking behavior and estimate each model assuming a cruise length of 20 and complete forward looking behavior. In addition, we estimate the models using a distance scale of 1, 10 and 20 to consider the role of travel costs in each model. For tables 10-14, each model is run for 400 Monte Carlo simulations and we estimate the welfare measures from equations (15) and (16), when location 1 is closed.

IV. Results

We have chosen three metrics of comparison for the base Monte Carlo simulations to investigate model fit: bias, root-mean squared error and within sample trajectory predictions. Bias and root-mean squared error are common metrics used in Monte Carlo estimation, whereas the within sample trajectory is utilized to determine how well the coefficients predict vessel dynamics within the sample.[15] For each iteration of the Monte Carlo the estimated parameter vector, $\hat{\boldsymbol{\beta}}$, was utilized to obtain a predicted path by selecting the location which possessed the highest probability of visitation in each time period as the site selected. Once a site was selected in time period t, the travel cost matrix, $\mathbf{x}_{\mathbf{t+1}}^{\mathbf{tc}}$, was altered to reflect the current site choice and then used to predict the site choice in time period $t+1$. This was conducted dynamically, so should a deviation occur at any time during the cruise an alternate cruise trajectory may be obtained.

Recall that the discount factor can not be estimated independently of the other parameters in the model. For all of our Monte Carlo runs, choices were generated with $\delta = 0.9$. We then estimate the DRUM assuming that the unknown (from the researcher's perspective) $\delta = 0.85$, 0.9, and 0.95. Focusing on the individual coefficients in the parameter vector it is evident that the dynamic model predominately possesses a lower bias and RMSE than the static model when looking across Tables 3-8. The bias and RMSE on the distance coefficient and the proxy for catch quality, β_0 and β_1, are always lower in the DRUM than the static model. This is not always true for the proxy on exogenous site selection information, β_2, where within the 10 location compact fishery model the static model slightly outperforms the DRUM. However, given the large bias and RMSE of the other coefficients and the predictive capabilities of the models, the static model is outperformed by the DRUM. The length of the cruise also compromises the performance of the static model, substantiating our first conjecture. For most of the Monte Carlo runs the bias and RMSE of the parameter vector in the static model increases with the length of the cruise, whereas in the DRUM model they remain relatively stable and in some cases even decrease. The magnitude of this effect is the greatest with the distance coefficient, β_0. This is most likely driven by the differences in the decision heuristics implied by the estimators. Since the static model assumes myopic haul-by-haul decision making it is forced to place all travel cost weight on the distance variable, whereas with the DRUM model travel cost is not only accounted for by the coefficient on distance in the current period, β_0, but also by the weight assigned to it in the value function.

Increasing the number of potential fishing sites also decreases the fit of the static model and increases the performance of the DRUM model, supporting our second conjecture. The only exception is for the 20 location model, where the bias and RMSE is lower than the 10 location model, yet still substantially larger than the 5 location model. Once again, these differences are the largest for the distance coefficient, β_0. Departures from the true discount factor ($\delta = 0.9$) increase the bias and RMSE and in particular for the distance coefficient, β_0. The degree of bias is most pronounced for the static model which is the extreme case, since $\delta = 0$. This result supports our third conjecture. As the cruise length increases, the relative predictive accuracy of the DRUM improves but only by extremely small margins. In terms of absolute predictive accuracy (denoted by Within %), the static model's performance decreases as the cruise length increases, whereas the DRUM model's performance marginally increases, or remains stable, for all the Monte Carlo runs. This highlights the importance of forward looking behavior in fisheries that have longer cruise lengths.

By far the most noticeable results from Tables 3-8 pertain to the geographic scale of the fishery. The static model's measures of bias, RMSE, and predictive accuracy are much closer to the DRUM when dealing with a spatially compact fishery (dist_scale=1) than with a large fishery (dist_scale=10). This supports the fourth conjecture we made regarding the performance of the DRUM model. Increasing the relative cost of switching areas demonstrates where the dynamic model outperforms the static model. The coefficients on the static model ($\delta = 0$) diverge from the true coefficients by as much as 50%. On the other hand, the DRUM coefficients closely track the true coefficients, usually diverging by less than 2% of the true estimate. Increasing the relative importance of distance also increased the predictive accuracy of the DRUM relative to the static model by as much as 46% (20 location model).

This relationship is perhaps better seen in Figures 4-7. Figures 4 and 5 depict the first 20 and last 20 hauls made on a 40 haul cruise assuming a distance scale of 1 for the 20 location model. Figures 6 and 7 depict the same information assuming a distance scale of 10. Both were randomly selected from the Monte Carlo simulations to compare the trajectories predicted by the static and DRUM estimators.[16] Figures 4 and 5 show trajectories for a compact fishery (dist_scale=1). Vessel behavior is very erratic because for each haul the vessel is selecting the site with the highest expected revenues without regard for the travel costs incurred because they are negligible. This causes both the static and the DRUM estimator to yield a poor prediction of site selection. In fact the static model slightly outperforms the DRUM estimator by accurately predicting 6 of the true sites whereas the DRUM estimator predicted only 4. However, when distance becomes relatively more important (Figure 6 and 7), the DRUM trajectory more closely tracks the observed choices, matching the cruise trajectory 37 of 40 hauls, as opposed to 16 of 40 hauls for the static model. These differences are readily evident in the cruise trajectories illustrated. For instance, the static model predicts that the vessel will be operating on the opposite end of the fishery for hauls 4 through 8 whereas the DRUM estimator closely matches the true trajectory. These differences will invariably not only yield erroneous fleet dynamics predictions but also biased welfare measures for site closures.

Differences in welfare measures, measuring the cost of area closures, across the static and dynamic models further magnify the importance of the geographic scale of the fishery. In Tables 10-14, we summarize the results from the Monte Carlo simulations investigating conjectures 5, 6 and 7. We do this for the spatially compact fishery (dist_scale=1), the large fishery (dist_scale=10), and a larger fishery (dist_scale=20) and assume varying degrees of asymmetry between the true time horizon employed by the fishermen versus that assumed by the researcher. Further we present CV estimates for both the spatially homogenous and spatially heterogeneous fishery (described in Table 9). Looking at the CV estimates, the static model nearly always predicts a higher amount of compensation than that from the dynamic model, this confirms our fifth conjecture. Since the dynamic model allows the agent to hedge future choices given a current choice, this should come as no surprise.

Comparing the true welfare measure to that predicted by equations (15) and (16), we can address our conjectures 6 and 7. When the true time horizon of the fisherman is 1 period ahead and the researcher assumes it is 20 the CV is better approximated by the static random utility model. Using the static welfare expressed in equation (15) the CV estimates within the homogeneous fishery are between 104.81% and 126.00% of the true, whereas they are between 97.5% and 44.97% for the DRUM estimation, equation (16). Within the heterogeneous fishery they are between 104.52% and 134.80% for the static and 105.72% and 53.07% for the DRUM. Aside from the spatially compact heterogeneous fishery (dist_scale =1), the DRUM consistently underestimates the true welfare impacts in this environment.

However, when the true time horizon is as small as 3 periods ahead, the DRUM welfare estimates are closer to the true welfare estimates even though the DRUM employed here assumes a time horizon of 20. For the homogeneous (heterogeneous) fishery the DRUM model is between 98.94% and 56.09% (106.97% and 66.24%) of the true welfare and the static model is between 106.46% and 178.27% (103.68% and 179.07%). Once again, the DRUM estimates are predominately lower than the true welfare impacts suggesting that in this setting our estimator may under estimate the welfare impacts of a closure, whereas the static model will overestimate them. When the true time horizon reaches one-half of that assumed by the researcher- a time horizon of 10 periods- the DRUM estimates of CV become much closer to the true welfare estimates, relative to the static model. This result holds for all three distance scales simulated. In addition, the magnitude of the over estimation generated by the static model increases with the distance scale. This suggests that in fisheries with high travel costs, policy makers should be concerned with the use of the static CV estimates provided that the cruise length is longer than a single day trip and fishermen possess some degree of forward looking behavior. Finally, when the true time horizon is equal to that assumed by the researcher, the DRUM estimates of CV are nearly identical to the true welfare impacts, further highlighting the importance of the DRUM estimator. These results support both conjecture 6 and 7. To summarize, our results suggest that when fishermen are suspected of being forward looking beyond one or two periods into the future, the researcher is better served using the DRUM with relatively long time horizons since the results are not overly sensitive to the choice of time horizon.

V. Conclusion

Location choices made by commercial fishermen are often made in a dynamic context; fishermen must decide where to fish from day to day while on a multi-day cruise. The literature on the economic tradeoffs associated with the spatial choice of commercial fishermen has all but ignored the dynamic nature of the fisherman's choice problem. The current choice of location spatially situates the vessel for the next period's location choice, and it is likely that agents examine how their current choice impacts future opportunities and payoffs while on a cruise. In this paper, we show that ignoring these dynamic linkages necessarily leads to biased parameter estimates and potentially bad policy guidance for many types of fisheries. Our results show that if agents are forward looking and the costs of switching areas are significant, the cost of spatial closures can be overestimated by a wide margin if static behavior is assumed. Further, an advantage of our model over the commonly used static model for welfare analysis is that the DRUM allows for the consistent aggregation of preferences across time in a multi-period discrete choice setting.

Our results also show that the computational burden of the DRUM model may not always be necessary. When the geographic scope of the fishery is small, when all areas are similar in terms of fishing productivity, or when known time horizons are no more than a few periods into the future, the DRUM provides only modest gains in terms of predictive accuracy, parameter estimates, and welfare measures even when choice data is generated assuming forward looking dynamic behavior.

Relative to the static model, the DRUM model we propose is computationally challenging especially when applied to problems with large numbers of choice alternatives and agents.

However, unlike dynamic models that have a non-deterministic state space, our model uses the same data and assumes the same information set used in a static discrete choice framework. Further, parameter and welfare estimates can usually be obtained in a matter of minutes. This allows our model to be computationally tractable even for a large number of choice alternatives and the model can be extended beyond problems having binary choices (Provencher 1995, Provencher and Bishop 1997, and Rust 1987) or only a few choice alternatives (Keane and Wolpin 1994, Smith and Provencher 2003). To the degree that location dominates the dynamic choice problem fishermen face, our model should perform well compared to a computationally more difficult model with more state dependence. However, as long as location is an important dynamic component, our model will likely outperform the commonly used static model, while remaining computationally tractable even with large numbers of choices and long time horizons.

The DRUM estimator can also be extended to other environmental resource problems where dynamics are likely to be important such as a wilderness backcountry trip (a sequence of discrete choices concerning camping or fishing locations), recreational angling, or international ecotourism trips, to name a few. In addition, given the importance of the time horizon assumption and the potential for heterogeneous vessel behavior, latent heterogeneity models can be used to answer a number of different questions regarding the forward looking behavior of fishermen as well as their corresponding discount factors. However, before the plethora of applications can be investigated, the relative performance of the DRUM estimator to the traditional static estimator needed to be established.

[1] This "sluggish" response parallels the theoretical work of Smith (1968,1969) and Clark (1980).

[2] For a more in depth and complete review of the application of discrete choice modeling to fisheries see Smith (2000).

[3] There are two exceptions to the static modeling paradigm (Curtis and Hicks 2000; Smith and Provencher 2003). Curtis and Hicks (2000) approximate a formal dynamic model by estimating the income stream from the duration of a cruise given a particular area selection. This approach, while *ad hoc* does allow for future considerations to impact current choice probabilities; however, we believe our approach by formalizing the dynamic linkages in the model allows for better estimates of the parameter vector and a consistent aggregation of preferences across time. Smith and Provencher (2003), utilize stochastic dynamic programming to investigate the role of information updating and the potential for search activity in the Northern California sea urchin fishery. They model behavior on a trip-by-trip basis, where a trip is a single day of fishing activity, whereas our model investigates dynamic behavior within a trip.

[4] All of these models are subject to the "curse of dimensionality" because the state space is assumed to be path dependent. Our model is only quasi-path dependent and assumes that fishermen know the spatial distribution of the relevant information prior to leaving port. This eliminates the "curse of dimensionality" and allows the researcher to dramatically expand the spatial resolution of the choice set and the duration of the cruise analyzed.

[5] Usual presentations of the model refer to the agent's objective function as a utility function. To keep our notation consistent throughout the paper, we refer to the objective function at location j in time t as R_{jt}.

[6] This section presents a special case of an estimable model derived by Keane and Wolpin. Wherever possible, we maintain their notation.

[7] Should these assumptions be deemed too restrictive alternative estimation techniques that employ simulation methods may be used to approximate the value function, for a more detailed discussion see Keane and Wolpin (1994). However, the extension of these methods to multiple site models may prove to be computationally intractable given more than a few choice alternatives.

[8] This follows the standard formulations derived by Hanemann 1984.

[9] Since measure 2 provides the lower bound welfare loss from the static model, we only report this measure in what follows.

[10] The maximum likelihood estimator was programmed in GAUSS and utilizes the constrained maximum likelihood (CML version 2.0) procedure. GAUSS code can be obtained from the corresponding author.

[11] Although the stylized solution algorithm appears to be a sequential operation, the value function calculation is nested in the log-likelihood calculation which implicitly solves the parameter vector and value function simultaneously. Therefore, this is a Full Information Maximum Likelihood estimator.

[12] The choice of starting values can increase the speed of the maximum likelihood estimation. Casual investigations in application of this model has illustrated that setting the starting values as a fraction, ie..

50%, of the static parameter values will help to increase the convergence speed of the estimator. However, within the Monte Carlos a starting value of 0 was used for all the parameter estimates.

[13] Comparing the models can be done utilizing the likelihood ratio static commonly used in econometrics with a χ_1^2 distribution on the test statistic.

[14] We experimented with other scale parameters (1.5, 2 and 2.5) and discovered that qualitative results are similar to those reported.

[15] Bias and root-mean squared error were calculated as follows, $Bias = \frac{1}{N}\sum_{i=1}^{N}(\beta_i - \hat{\beta}_i)$ and

$RMSE = sqrt\left(\frac{1}{N}\sum_{i=1}^{N}\left(\beta_i - \hat{\beta}_i\right)^2\right)$ where N is the number of Monte Carlo runs (500 for each model).

[16] Our estimator places vessels at zone centerpoints only (corresponding to Figure 3). Predicted points in Figures 4-7 were randomly perturbed slightly to avoid stacking spatial information.

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Table 1: Distributional assumptions of the data (compact fishery).

	5 – Location		10 – Location		20 – Location	
	μ	σ	μ	σ	μ	σ
$X_{1,1}$	10	1	10	1	10.5	1
$X_{1,2}$	10	1	10.5	1	10	1
$X_{1,3}$	9.5	1	9.5	1	9	1
$X_{1,4}$	10.5	1	10	1	10	1
$X_{1,5}$	10	1	10	1	10	1
$X_{1,6}$	---	---	10	1	10.5	1
$X_{1,7}$	---	---	10	1	10	1
$X_{1,8}$	---	---	9.5	1	9.5	1
$X_{1,9}$	---	---	10	1	9.5	1
$X_{1,10}$	---	---	10	1	10	1
$X_{1,11}$	---	---	---	---	10.5	1
$X_{1,12}$	---	---	---	---	10	1
$X_{1,13}$	---	---	---	---	9.5	1
$X_{1,14}$	---	---	---	---	10	1
$X_{1,15}$	---	---	---	---	10	1
$X_{1,16}$	---	---	---	---	10.5	1
$X_{1,17}$	---	---	---	---	10	1
$X_{1,18}$	---	---	---	---	9.5	1
$X_{1,19}$	---	---	---	---	10	1
$X_{1,20}$	---	---	---	---	10	1
$X_{2,1}$	5	2	5	2	5	2
$X_{2,2}$	5	2	5	2	5	2
$X_{2,3}$	5	2	5	2	5	2
$X_{2,4}$	5	2	5	2	5	2
$X_{2,5}$	5	2	5	2	5	2
$X_{2,6}$	---	---	5	2	5	2
$X_{2,7}$	---	---	5	2	5	2
$X_{2,8}$	---	---	5	2	5	2
$X_{2,9}$	---	---	5	2	5	2
$X_{2,10}$	---	---	5	2	5	2
$X_{2,11}$	---	---	---	---	5	2
$X_{2,12}$	---	---	---	---	5	2
$X_{2,13}$	---	---	---	---	5	2
$X_{2,14}$	---	---	---	---	5	2
$X_{2,15}$	---	---	---	---	5	2
$X_{2,16}$	---	---	---	---	5	2
$X_{2,17}$	---	---	---	---	5	2
$X_{2,18}$	---	---	---	---	5	2
$X_{2,19}$	---	---	---	---	5	2
$X_{2,20}$	---	---	---	---	5	2

Table 2: Distributional assumptions of the data (large fishery).

	5 – Location		10 – Location		20 – Location	
	μ	σ	μ	σ	μ	σ
$X_{1,1}$	120	40	120	40	110	40
$X_{1,2}$	100	10	100	20	100	20
$X_{1,3}$	100	10	120	40	100	20
$X_{1,4}$	120	40	100	20	100	20
$X_{1,5}$	110	20	100	20	100	20
$X_{1,6}$	---	---	100	20	110	40
$X_{1,7}$	---	---	100	20	100	20
$X_{1,8}$	---	---	120	40	100	20
$X_{1,9}$	---	---	100	20	100	20
$X_{1,10}$	---	---	120	40	100	20
$X_{1,11}$	---	---	---	---	100	20
$X_{1,12}$	---	---	---	---	120	40
$X_{1,13}$	---	---	---	---	100	20
$X_{1,14}$	---	---	---	---	100	20
$X_{1,15}$	---	---	---	---	100	20
$X_{1,16}$	---	---	---	---	100	20
$X_{1,17}$	---	---	---	---	100	20
$X_{1,18}$	---	---	---	---	100	20
$X_{1,19}$	---	---	---	---	100	20
$X_{1,20}$	---	---	---	---	120	40
$X_{2,1}$	50	20	50	20	50	20
$X_{2,2}$	45	20	50	20	50	20
$X_{2,3}$	45	20	50	20	50	20
$X_{2,4}$	50	20	50	20	50	20
$X_{2,5}$	50	20	50	20	50	20
$X_{2,6}$	---	---	50	20	50	20
$X_{2,7}$	---	---	50	20	50	20
$X_{2,8}$	---	---	50	20	50	20
$X_{2,9}$	---	---	50	20	50	20
$X_{2,10}$	---	---	50	20	50	20
$X_{2,11}$	---	---	---	---	50	20
$X_{2,12}$	---	---	---	---	50	20
$X_{2,13}$	---	---	---	---	50	20
$X_{2,14}$	---	---	---	---	50	20
$X_{2,15}$	---	---	---	---	50	20
$X_{2,16}$	---	---	---	---	50	20
$X_{2,17}$	---	---	---	---	50	20
$X_{2,18}$	---	---	---	---	50	20
$X_{2,19}$	---	---	---	---	50	20
$X_{2,20}$	---	---	---	---	50	20

Table 3: 5 Location Model with dist_scale=1.

5 Location Model	β_0 Bias	β_0 RMSE	β_1 Bias	β_1 RMSE	β_2 Bias	β_2 RMSE	Within %
Cruise = 10							
Static; $\delta=0$	0.01041	0.02201	0.00041	0.01799	-0.0042	0.01290	45.22%
Dynamic; $\delta=0.85$	0.00448	0.01858	-0.0012	0.01775	0.00056	0.01207	46.20%
Dynamic; $\delta=0.9$	-0.0001	0.01785	-0.0008	0.01773	-0.0002	0.01203	46.26%
Dynamic; $\delta=0.95$	-0.0049	0.01832	-0.0005	0.01768	-0.0009	0.01205	46.33%
Cruise = 20							
Static; $\delta=0$	0.01088	0.02174	0.00013	0.01818	-0.0046	0.01304	45.17%
Dynamic; $\delta=0.85$	0.00474	0.01820	-0.0010	0.01783	0.00052	0.01209	46.19%
Dynamic; $\delta=0.9$	0.00016	0.01739	-0.0007	0.01778	-0.0002	0.01205	46.24%
Dynamic; $\delta=0.95$	-0.0047	0.01784	-0.0004	0.01774	-0.0011	0.01208	46.31%
Cruise = 40							
Static; $\delta=0$	0.01125	0.02197	0.00053	0.01829	-0.0049	0.01333	45.05%
Dynamic; $\delta=0.85$	0.00488	0.01823	-0.0008	0.01787	0.00040	0.01229	46.14%
Dynamic; $\delta=0.9$	0.00028	0.01738	-0.0005	0.01781	-0.0004	0.01226	46.20%
Dynamic; $\delta=0.95$	-0.0002	0.01782	-0.0002	0.01776	-0.0012	0.01229	46.27%

Table 4: 10 Location Model with dist_scale=1.

10 Location Model	β_0 Bias	β_0 RMSE	β_1 Bias	β_1 RMSE	β_2 Bias	β_2 RMSE	Within %
Cruise = 10							
Static; $\delta=0$	0.02611	0.03979	-0.0025	0.01700	0.00030	0.01020	32.23%
Dynamic; $\delta=0.85$	0.00497	0.02851	-0.0006	0.01681	0.00088	0.01025	32.73%
Dynamic; $\delta=0.9$	0.00045	0.02786	-0.0004	0.01680	0.00070	0.01024	32.75%
Dynamic; $\delta=0.95$	-0.0043	0.02796	-0.0003	0.01680	0.00051	0.01022	32.77%
Cruise = 20							
Static; $\delta=0$	0.03135	0.04350	-0.0028	0.01707	0.00017	0.00998	32.30%
Dynamic; $\delta=0.85$	0.00527	0.02813	-0.0005	0.01684	0.00094	0.01005	32.77%
Dynamic; $\delta=0.9$	0.00051	0.02734	-0.0003	0.01603	0.00076	0.01004	32.79%
Dynamic; $\delta=0.95$	-0.0045	0.02751	-0.0002	0.01683	0.00058	0.01002	34.56%
Cruise = 40							
Static; $\delta=0$	0.03396	0.04540	-0.0028	0.01755	0.00017	0.00999	32.29%
Dynamic; $\delta=0.85$	0.00530	0.02803	-0.0005	0.01692	0.00101	0.01008	32.78%
Dynamic; $\delta=0.9$	0.00042	0.02728	-0.0003	0.01691	0.00084	0.01006	32.80%
Dynamic; $\delta=0.95$	-0.0047	0.02743	-0.0002	0.01691	0.00065	0.01005	32.81%

Table 5: 20 Location Model with dist_scale=1.

20 Location Model	β_0		β_1		β_2		Within
	Bias	RMSE	Bias	RMSE	Bias	RMSE	%
Cruise = 10							
Static; $\delta=0$	0.01672	0.02341	0.02611	0.03039	-0.0030	0.00994	20.06%
Dynamic; $\delta=0.85$	0.00322	0.01623	0.00124	0.01547	-0.0002	0.00956	21.18%
Dynamic; $\delta=0.9$	0.00000	0.01582	0.00007	0.01541	-0.0005	0.00957	21.22%
Dynamic; $\delta=0.95$	-0.0035	0.01610	-0.0011	0.01544	-0.0008	0.00959	21.26%
Cruise = 20							
Static; $\delta=0$	0.01864	0.02463	0.02779	0.03185	-0.0030	0.00996	19.91%
Dynamic; $\delta=0.85$	0.00336	0.01521	0.00152	0.01543	-0.0002	0.00932	21.12%
Dynamic; $\delta=0.9$	-0.0000	0.01535	0.00027	0.01534	-0.0005	0.00951	21.17%
Dynamic; $\delta=0.95$	-0.0037	0.01567	-0.0009	0.01536	-0.0008	0.00954	21.20%
Cruise = 40							
Static; $\delta=0$	0.01926	0.02491	0.02886	0.03279	-0.0030	0.00981	19.88%
Dynamic; $\delta=0.85$	0.00328	0.01544	0.00175	0.01544	-0.0002	0.00936	21.14%
Dynamic; $\delta=0.9$	-0.0002	0.01499	0.00045	0.01534	-0.0005	0.00937	21.17%
Dynamic; $\delta=0.95$	-0.0047	0.01782	-0.0002	0.01777	-0.0012	0.01230	21.19%

Table 6: 5 Location Model with dist_scale =10.

5 Location Model	β_0		β_1		β_2		Within
	Bias	RMSE	Bias	RMSE	Bias	RMSE	%
Cruise = 10							
Static; $\delta=0$	-0.3132	0.31318	0.13217	0.13218	-0.3298	0.32980	76.44%
Dynamic; $\delta=0.85$	0.01267	0.02744	-0.0006	0.00983	0.00229	0.02432	93.30%
Dynamic; $\delta=0.9$	0.00226	0.02378	-0.0010	0.00984	0.00233	0.02434	93.54%
Dynamic; $\delta=0.95$	-0.0226	0.03132	0.00434	0.01019	-0.0116	0.02555	93.54%
Cruise = 20							
Static; $\delta=0$	-0.3147	0.31469	0.13328	0.13328	-0.3336	0.33362	75.13%
Dynamic; $\delta=0.85$	0.01231	0.02738	-0.0007	0.00981	0.00186	0.02433	93.38%
Dynamic; $\delta=0.9$	0.00211	0.02364	-0.0009	0.00968	0.00206	0.02399	93.66%
Dynamic; $\delta=0.95$	-0.0232	0.03139	0.00486	0.01013	-0.0125	0.02531	93.60%
Cruise = 40							
Static; $\delta=0$	-0.3153	0.31533	0.13382	0.13382	-0.3349	0.33491	75.07%
Dynamic; $\delta=0.85$	0.01195	0.02717	-0.0002	0.00984	0.00135	0.02438	93.33%
Dynamic; $\delta=0.9$	0.00166	0.02360	-0.0007	0.00976	0.00158	0.02409	93.68%
Dynamic; $\delta=0.95$	-0.0239	0.03191	0.00488	0.01024	-0.0133	0.02582	93.65%

Table 7: 10 Location Model with dist_scale=10.

10 Location Model	β_0 Bias	β_0 RMSE	β_1 Bias	β_1 RMSE	β_2 Bias	β_2 RMSE	Within %
Cruise = 10							
Static; δ=0	-0.2481	0.24819	0.10559	0.10560	-0.2559	0.25596	78.32%
Dynamic; δ=0.85	0.01114	0.02193	-0.0002	0.00741	0.00105	0.01827	91.10%
Dynamic; δ=0.9	0.00255	0.01888	-0.0005	0.00753	0.00268	0.01854	91.30%
Dynamic; δ=0.95	-0.0112	0.02118	-0.0001	0.00737	-0.0002	0.01812	91.35%
Cruise = 20							
Static; δ=0	-0.2514	0.25141	0.10723	0.10725	-0.2582	0.25819	76.78%
Dynamic; δ=0.85	0.01174	0.02296	-0.0004	0.00776	0.00154	0.01893	91.07%
Dynamic; δ=0.9	0.00254	0.01953	-0.0009	0.00780	0.00249	0.01898	91.19%
Dynamic; δ=0.95	-0.0120	0.02214	0.00045	0.00759	-0.0014	0.01846	91.26%
Cruise = 40							
Static; δ=0	-0.2546	0.25466	0.10860	0.10861	-0.2615	0.26151	76.64%
Dynamic; δ=0.85	0.01192	0.02262	-0.0004	0.00763	0.00162	0.01869	91.09%
Dynamic; δ=0.9	0.00218	0.01883	-0.0008	0.00762	0.00219	0.01860	91.26%
Dynamic; δ=0.95	-0.0136	0.02215	0.00078	0.00743	-0.0022	0.01811	91.28%

Table 8: 20 Location Model with dist_scale=10.

20 Location Model	β_0 Bias	β_0 RMSE	β_1 Bias	β_1 RMSE	β_2 Bias	β_2 RMSE	Within %
Cruise = 10							
Static; δ=0	-0.2844	0.28440	0.12075	0.12076	-0.2972	0.29722	61.63%
Dynamic; δ=0.85	0.00859	0.01919	0.00029	0.00694	-2.1E-5	0.01661	87.39%
Dynamic; δ=0.9	0.00248	0.01711	-0.0009	0.00702	0.00247	0.01681	88.01%
Dynamic; δ=0.95	-0.0188	0.02441	0.00333	0.00737	-0.0091	0.01807	88.04%
Cruise = 20							
Static; δ=0	-0.2877	0.28775	0.12223	0.12223	-0.2987	0.29874	59.97%
Dynamic; δ=0.85	0.00826	0.01944	0.00020	0.00696	-0.0004	0.01698	87.37%
Dynamic; δ=0.9	0.00191	0.01766	-0.0007	0.00708	0.00177	0.01728	88.13%
Dynamic; δ=0.95	-0.0199	0.02565	0.00402	0.00782	-0.0095	0.01890	88.12%
Cruise = 40							
Static; δ=0	-0.2895	0.28947	0.12281	0.12282	-0.2998	0.29985	60.04%
Dynamic; δ=0.85	0.00842	-0.0194	-0.0001	0.00693	-0.0002	0.01675	87.30%
Dynamic; δ=0.9	0.00212	0.01799	-0.0008	0.00697	0.00214	0.01693	88.04%
Dynamic; δ=0.95	-0.0202	0.02568	0.00430	0.00778	-0.0096	0.01851	88.04%

Table 9. Data generation for Welfare Results.

	5 – Location Homogenous Fishery		5 – Location Heterogeneous Fishery	
	μ	σ	μ	σ
$X_{1,1}$	110	20	80	20
$X_{1,2}$	110	20	60	40
$X_{1,3}$	110	20	60	40
$X_{1,4}$	110	20	60	40
$X_{1,5}$	110	20	80	20

Table 10. Welfare results: True time horizon = 1.

Time Horizon – 1 period	Bias β_0	Bias β_1	RMSE β_0	RMSE β_1	Within	True Welfare	Static Welf.	Dynamic Welf.
Homogeneous								
Dyn(dist=1)	-0.0040	-0.0004	0.03903	0.01803	91.99%	23.191	-----	22.622
Stat(dist=1)	-0.0212	0.02216	0.04999	0.02758	91.49%	23.191	24.306	-----
Dyn(dist=10)	-0.2430	0.20878	0.24304	0.20891	82.31%	22.482	-----	15.691
Stat(dist=10)	-0.3395	0.32849	0.33947	0.32850	75.07%	22.482	28.910	-----
Dyn(dist=20)	-0.3720	0.35921	0.37205	0.35922	64.59%	27.097	-----	12.185
Stat(dist=20)	-0.3971	0.39287	0.39705	0.39288	58.97%	27.097	34.144	-----
Heterogeneous								
Dyn(dist=1)	-0.0037	-0.0009	0.04922	0.02258	94.62%	34.266	-----	36.226
Stat(dist=1)	0.01309	0.01806	0.06455	0.02716	94.36%	34.266	35.100	-----
Dyn(dist=10)	-0.1908	0.17005	0.19112	0.17038	89.56%	33.545	-----	24.881
Stat(dist=10)	-0.3365	0.34055	0.33648	0.34056	81.97%	33.545	46.475	-----
Dyn(dist=20)	-0.3567	0.33626	0.35669	0.33627	75.91%	41.760	-----	22.162
Stat(dist=20)	-0.4043	0.40554	0.40426	0.40554	69.90%	41.760	56.293	-----

Table 11. Welfare results: True time horizon = 3.

Time Horizon – 3 periods	Bias β_0	Bias β_1	RMSE β_0	RMSE β_1	Within	True Welfare	Static Welf.	Dynamic Welf.
Homogeneous								
Dyn(dist=1)	-0.0007	-0.0005	0.03894	0.01811	91.99%	22.836	-----	22.617
Stat(dist=1)	-0.0167	0.02222	0.04802	0.02767	91.49%	22.836	24.311	-----
Dyn(dist=10)	-0.0556	0.05289	0.05807	0.05554	90.66%	18.502	-----	14.700
Stat(dist=10)	-0.3154	0.32758	0.31560	0.32759	73.20%	18.502	29.288	-----
Dyn(dist=20)	-0.2510	0.24269	0.25111	0.24284	76.56%	19.450	-----	10.910
Stat(dist=20)	-0.3883	0.39641	0.38828	0.39641	55.77%	19.450	34.674	-----
Heterogeneous								
Dyn(dist=1)	-0.0024	-0.0010	0.04902	0.02263	94.61%	33.860	-----	36.221
Stat(dist=1)	0.01394	0.01810	0.06432	0.02726	94.36%	33.860	35.106	-----
Dyn(dist=10)	-0.0082	0.00607	0.02501	0.02469	94.50%	28.586	-----	24.247
Stat(dist=10)	-0.3200	0.33717	0.31999	0.33718	80.76%	28.586	46.857	-----
Dyn(dist=20)	-0.1880	0.18059	0.18849	0.18109	87.73%	31.703	-----	21.00
Stat(dist=20)	-0.3899	0.40385	0.38986	0.40385	66.66%	31.703	56.771	-----

Table 12. Welfare results: True time horizon = 5.

Time Horizon – 5 periods	Bias β_0	Bias β_1	RMSE β_0	RMSE β_1	Within	True Welfare	Static Welf.	Dynamic Welf.
Homogeneous								
Dyn(dist=1)	-0.0007	-0.0005	0.03894	0.01811	91.99%	22.744	-----	22.615
Stat(dist=1)	-0.0167	0.02222	0.04801	0.02767	91.49%	22.744	24.311	-----
Dyn(dist=10)	-0.0055	0.00466	0.02237	0.02239	92.53%	17.145	-----	14.642
Stat(dist=10)	-0.3139	0.32694	0.31386	0.32695	72.93%	17.145	29.289	-----
Dyn(dist=20)	-0.1276	0.12562	0.12877	0.12682	85.70%	16.428	-----	10.628
Stat(dist=20)	-0.3849	0.39756	0.38494	0.39756	53.32%	16.428	35.098	-----
Heterogeneous								
Dyn(dist=1)	-0.0024	-0.0010	0.04902	0.02263	94.61%	33.722	-----	36.227
Stat(dist=1)	0.01394	0.01809	0.06432	0.02726	94.35%	33.722	35.200	-----
Dyn(dist=10)	-0.0020	0.00085	0.02365	0.02392	94.63%	26.719	-----	24.229
Stat(dist=10)	-0.3195	0.33748	0.31986	0.33748	80.55%	26.719	46.907	-----
Dyn(dist=20)	-0.4604	0.04247	0.05276	0.04977	94.32%	28.355	-----	20.853
Stat(dist=20)	-0.3871	0.40322	0.38714	0.40322	65.85%	28.355	57.074	-----

Table 13. Welfare results: True time horizon = 10.

Time Horizon – 10 periods	Bias β_0	Bias β_1	RMSE β_0	RMSE β_1	Within	True Welfare	Static Welf.	Dynamic Welf.
Homogeneous								
Dyn(dist=1)	-0.0007	-0.0005	0.03894	0.01811	91.99%	22.662	-----	22.615
Stat(dist=1)	-0.0167	0.02222	0.04801	0.02767	91.49%	22.662	24.311	-----
Dyn(dist=10)	0.00234	-0.0025	0.02249	0.02269	92.85%	15.494	-----	14.629
Stat(dist=10)	-0.3134	0.32695	0.31346	0.32696	72.88%	15.494	29.296	-----
Dyn(dist=20)	-0.0041	0.00460	0.03055	0.03105	93.15%	12.477	-----	10.548
Stat(dist=20)	-0.3815	0.39386	0.38151	0.39586	53.21%	12.477	34.913	-----
Heterogeneous								
Dyn(dist=1)	-0.0024	-0.0010	0.04902	0.02263	94.61%	33.627	-----	36.227
Stat(dist=1)	0.01394	0.01809	0.06432	0.02726	94.35%	33.627	35.100	-----
Dyn(dist=10)	0.00237	-0.0026	0.02426	0.02457	94.76%	25.012	-----	24.215
Stat(dist=10)	-0.3194	0.33748	0.31944	0.33748	80.52%	25.012	46.931	-----
Dyn(dist=20)	0.00346	-0.0037	0.03279	0.03277	95.46%	23.190	-----	20.805
Stat(dist=20)	-0.3858	0.40287	0.38576	0.40287	65.48%	23.190	57.038	-----

Table 14. Welfare results: True time horizon = 20.

Time Horizon – 20 periods	Bias β_0	Bias β_1	RMSE β_0	RMSE β_1	Within	True Welfare	Static Welf.	Dynamic Welf.
Homogeneous								
Dyn(dist=1)	-0.0007	-0.0005	0.03894	0.01811	91.99%	22.605	-----	22.615
Stat(dist=1)	-0.0167	0.02222	0.04801	0.02767	91.49%	22.605	24.311	-----
Dyn(dist=10)	0.00234	-0.0025	0.02249	0.02268	92.85%	14.625	-----	14.629
Stat(dist=10)	-0.3134	0.32695	0.31346	0.32697	72.88%	14.625	29.296	-----
Dyn(dist=20)	0.00484	-0.0045	0.03155	0.03187	93.40%	10.549	-----	10.548
Stat(dist=20)	-0.3815	0.39576	0.38145	0.39576	53.25%	10.549	34.912	-----
Heterogeneous								
Dyn(dist=1)	-0.0024	-0.0010	0.04902	0.02263	94.61%	33.559	-----	33.577
Stat(dist=1)	0.01394	0.01809	0.06432	0.02726	94.35%	33.559	35.100	-----
Dyn(dist=10)	0.00234	-0.0026	0.02424	0.02450	94.76%	24.205	-----	24.215
Stat(dist=10)	-0.3194	0.33748	0.31944	0.33748	80.53%	24.205	46.931	-----
Dyn(dist=20)	0.00393	-0.0041	0.03266	0.03264	95.47%	20.799	-----	20.804
Stat(dist=20)	-0.3858	0.40289	0.38575	0.40287	65.48%	20.799	57.040	-----

Figure 1: 5 Location Spatial Layout

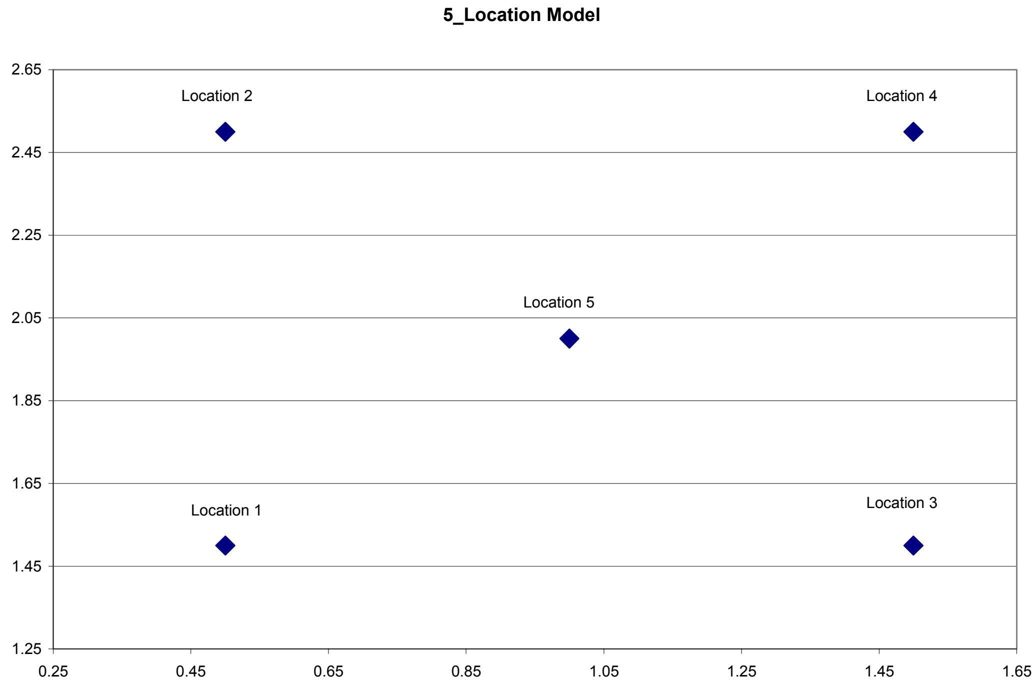

Figure 2: 10 Location Spatial Layout

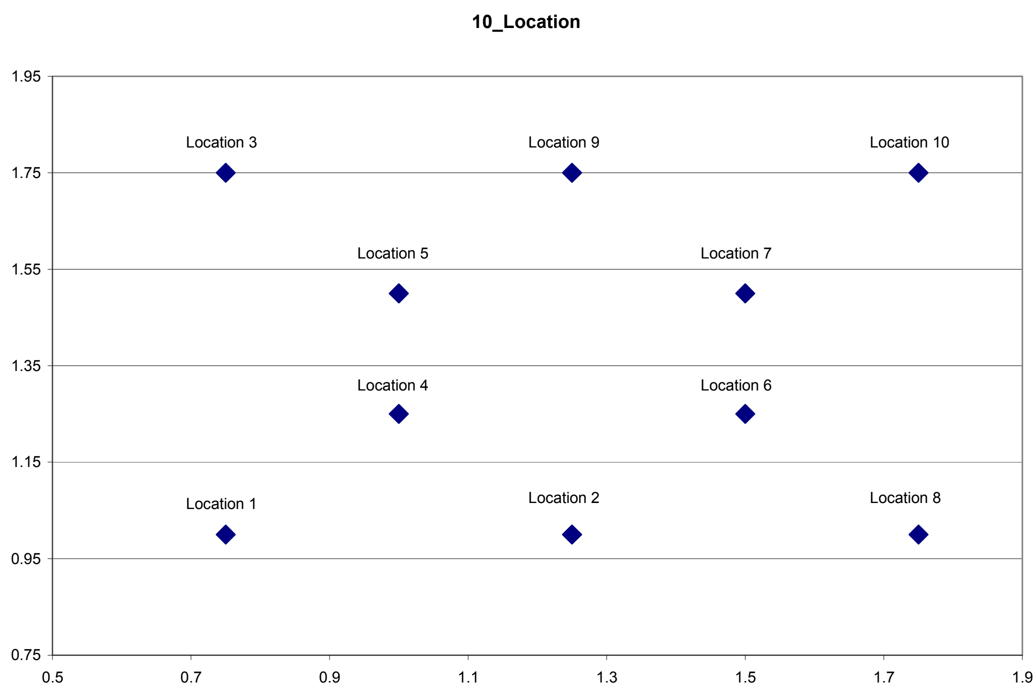

Figure 3: 20 Location Spatial Layout

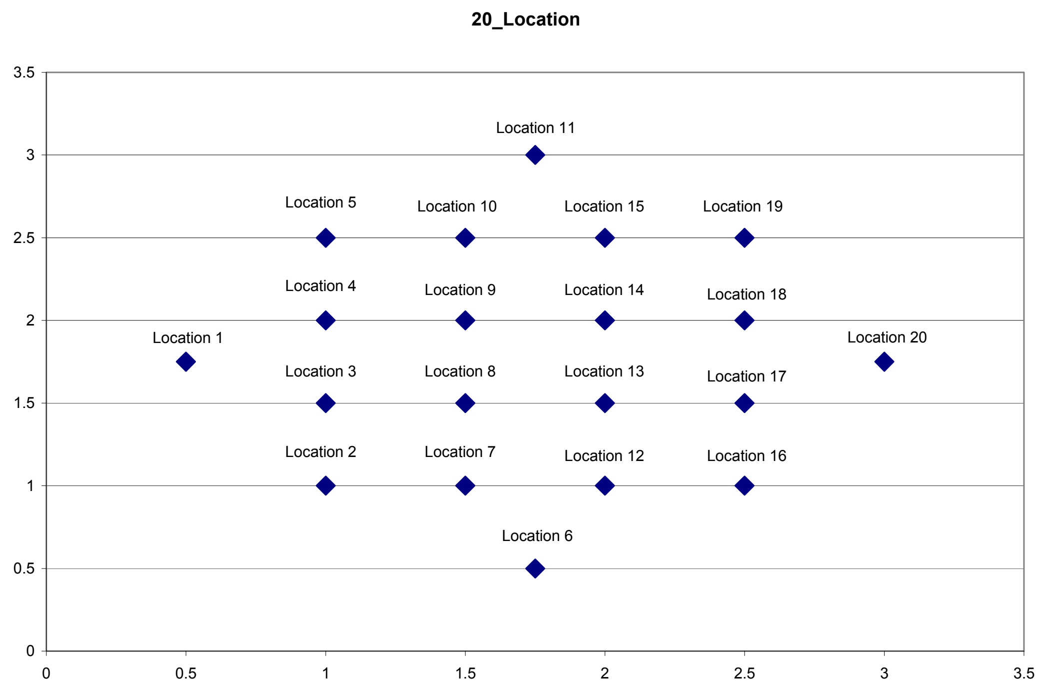

Figure 4: Hauls 1-20 for the 20 location model with dist_scale=1 (left is the static model and right is the DRUM estimator).

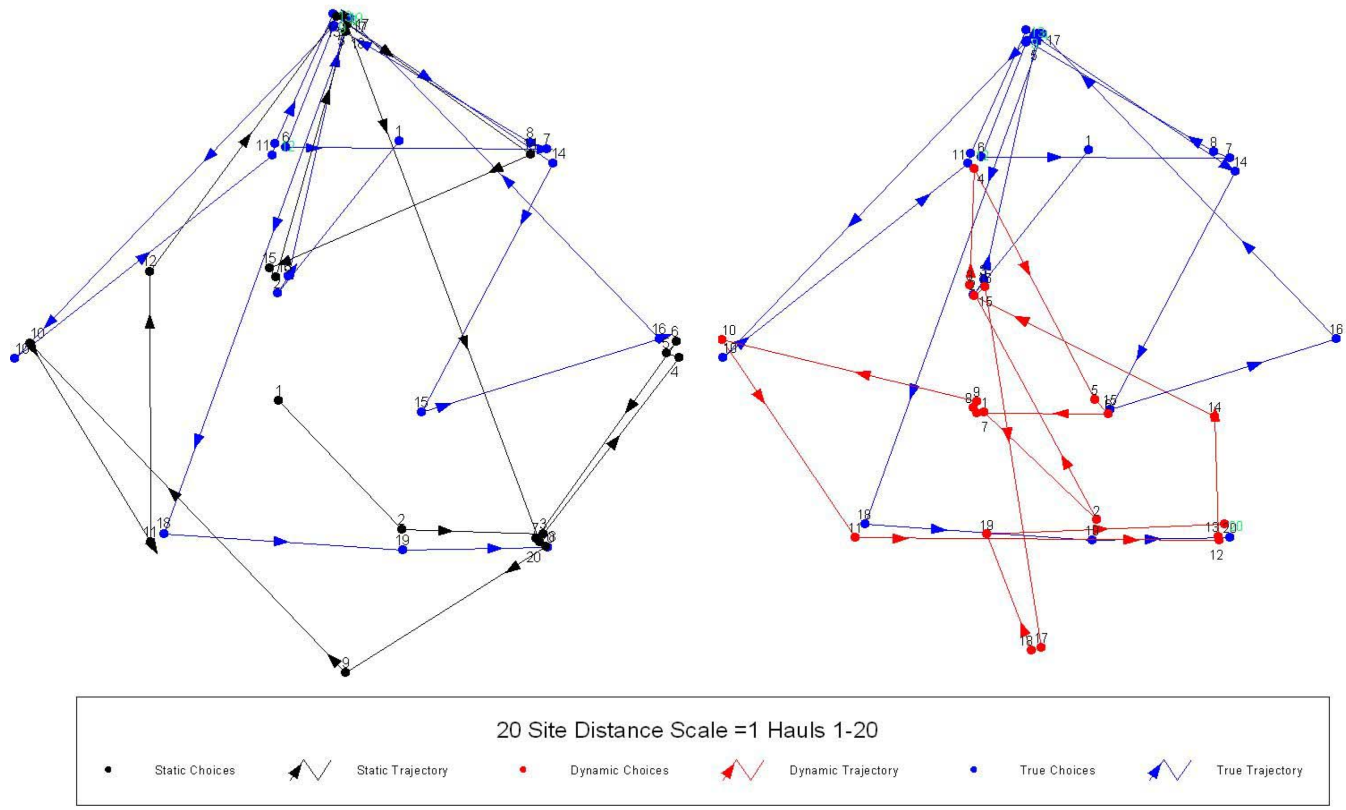

Figure 5: Hauls 21-40 for the 20 location model with dist_scale=1(left is the static model and right is the DRUM estimator).

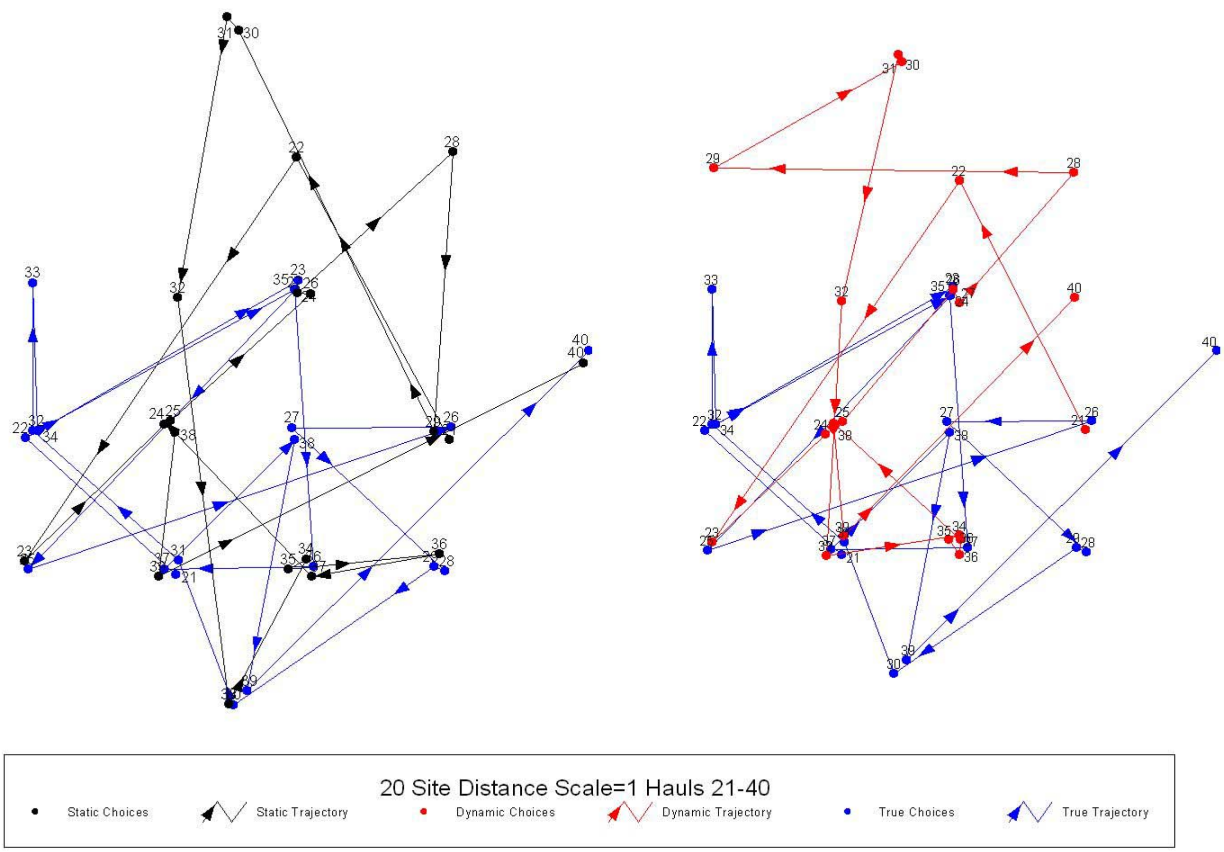

Figure 6: Hauls 1-20 for the 20 location model with dist_scale=10(left is the static model and right is the DRUM estimator).

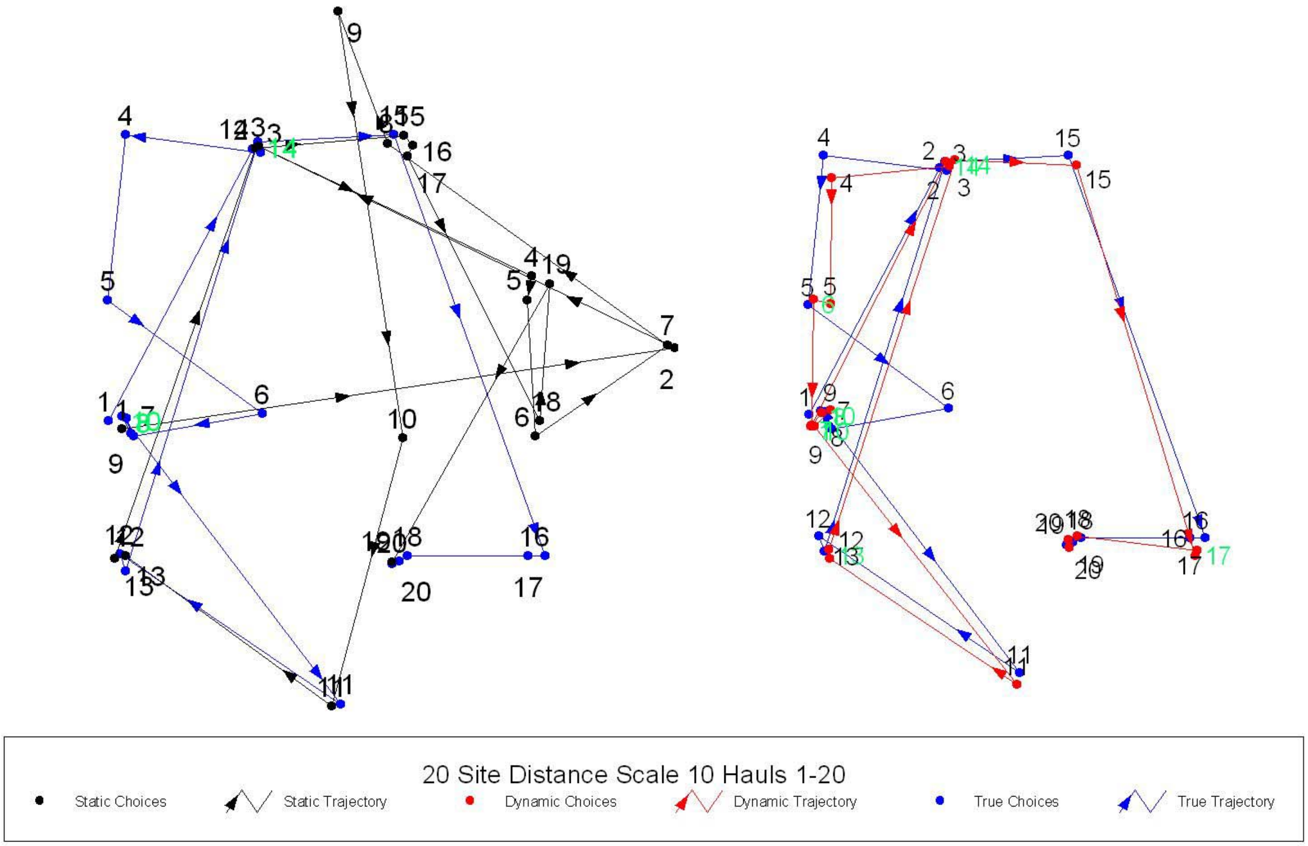

Figure 7: Hauls 21-40 for the 20 location model with dist_scale=10(left is the static model and right is the DRUM estimator).