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Affirmative Action, Incentives and the Black–White Test Score Gap

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Abstract

This paper develops a theoretical model of college admissions to study the effects of affirmative action policies on the high school achievement of college bound students. The innovation is to include endogenous human capital decisions in the model. When colleges switch admissions policies, they implicitly alter the likelihood of acceptance earned by a given human capital investment. Thus, human capital investments are sensitive to changes in admissions policies. The main results are that banning affirmative action increases the black–white test score gap and decreases college enrollment and social welfare of the minority group.

JEL Classification: I20, J71, D82.

Keywords: Affirmative Action, Discrimination, Public Policy, Education, Asymmetric Information

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> "An applicant's LSAT score can improve dramatically with preparation, but such preparation is a cost, and there must be sufficient benefits attached to an improved score to justify the additional cost...As admissions prospects near certainty, there is no incentive for the black applicant to continue to prepare for the LSAT once he is reasonably assured of achieving the requisite score...
>
> Indeed, the very existence of racial discrimination of the type practiced by the Law School may impede the narrowing of the test score gap."
>
> —Justice Thomas, Opinion in *Grutter v. Bollinger, et al.*, 2003, p. 30.

1 Introduction

Justice Thomas argues in his *Grutter* opinion that preferential college admissions policies undermine the incentives for blacks to prepare for the LSAT (Law School Admissions Test). His argument is that less stringent admissions requirements erode the incentive to achieve. While this may be true for the brightest applicants, I assert that arguments of this type do not recognize that affirmative action creates positive incentives for other minority applicants. Affirmative action policies are designed to increase educational opportunities for minorities, by relaxing admissions standards for the targeted group. When colleges and universities use affirmative action, many minority applicants should perceive a higher marginal return to preparatory education. By this argument, these students should allocate greater resources to academic success.

The concept of incentives is essential to this paper. In this paper, I present a model of college admissions in which I formulate the test preparation discussed above as human capital investments made by high school students applying to college. By specifying a model in which human capital investments are endogenous, I am able to analyze how changing admissions policies differentially affects minorities' and non–minorities' human capital formation during secondary school and how the test score gap between minorities and non–minorities changes as a result.

My theory allows me to study the distributional as well as the aggregate changes in human capital investment resulting from changes in admissions policy. Evaluating the distributional effects is important because, as discussed above and alluded to by Justice

Thomas, changing admissions policies differentially affects the incentives to invest for individuals within a racial group. For example, my theory predicts that in response to a ban on affirmative action, high ability minorities increase human capital investment while low and middle ability minorities decrease human capital investment, consistent with the intuitive description of behavior given above. Specifying social welfare as an increasing concave function of human capital investment, I show that banning affirmative action decreases minorities' social welfare. Thus I am able to evaluate the welfare effects of policy changes that differentially affect individuals within a given racial group.

That the incentives put forth by a particular admissions policy affect college applicants' behavior has been largely ignored by the economics literature until Long (2002a) and Fryer, Loury and Yuret (2003).[1] Other discussions regarding prohibition of affirmative action, such as Chan and Eyster (2003) and Epple, Romano and Sieg (2003) take test scores to be deterministic. Thus, they are unable to evaluate the importance of incentives when discussing prohibition of affirmative action, and cannot comment on how or why banning affirmative action may affect test score differentials between minority and non–minority high school students.

Understanding the test score gap between blacks and whites is fundamental to achieving an equal society. As stated by Jencks and Phillips (1998), "if racial equality is America's goal, reducing the black–white test score gap would probably do more to promote this goal than any other strategy that commands broad political support. Reducing the test score gap is probably both necessary and sufficient for substantially reducing racial inequality in educational attainment and earnings." (p. 4) The test score gap has closed over time; Hedges and Nowell (1998) report that blacks' scores on a test of composite achievement were, on average, 1.18 standard deviations below whites' in 1965, and .82 standard deviations below in 1992.[2] My results suggest that a portion of the narrowing of the gap can be attributed to improved incentives for minorities over time via implementation of affirmative action programs and increased minority access to higher education. This carries the implication that banning affirmative action could

[1]Fryer et al. (2003) study a model similar to the one presented here, developed concurrently and independently.

[2]The standard errors for these estimates are .020 and .107; thus they are highly significant.

reverse some of the gains.

The existence of the test score gap is the underlying reason colleges and universities continue to use affirmative action in admissions, since race blind policies would not admit sufficient minorities to maintain campus diversity. Banning affirmative action would likely decrease minority enrollment, as has happened in Texas and California, since a ban would effectively force minority applicants to meet more stringent admissions requirements. For example, The UT–Austin Office of Institutional Research reports that fall enrollment of black first time freshmen fell from 266 in 1996 to 190 in 1997, the first year after the *Hopwood* decision prohibited affirmative action.[3][4] Similarly, the UC–Berkeley Office of Student Research reports that fall enrollment of black first time freshmen fell from 1270 in 1997 to 1159 in 1998, the first year after the Board of Regents voted to end affirmative action.[5]

Long (2002a) shows empirically that in response to bans on affirmative action, minority students in Texas and California sent their SAT scores to less selective post–secondary institutions, while non–minorities sent their SAT scores to more selective institutions. Thus, some of the enrollment effects mentioned above can be attributed to shifts in application behavior. My results suggest that the human capital investment response of the applicants is responsible for an additional portion of these enrollment changes. While some of the changes in enrollment are a direct result of test score differentials between minorities and non–minorities, there is also an indirect feedback effect through which minority achievement is eroded. Ending race conscious admissions practices decreases minorities' incentive to invest in human capital, thereby increasing the test score gap itself and further decreasing minority enrollment.

Additionally, I study the implications of banning affirmative action for college quality. I assume that colleges maximize an index of student quality that increases with applicants' investments and exhibits positive returns to diversity. Banning affirmative action results in an equilibrium in which human capital investments are more productive for the marginally admitted minority applicant than the marginally admitted non–minority

[3] *Cheryl J. Hopwood, et al. v. State of Texas.*

[4] See http://www.utexas.edu/academic/oir/statistical_handbook/02-03/students/s12a/.

[5] See http://osr.berkeley.edu/public/staffweb/tc/trends/ethf01tb1aver1.html.

applicant. This occurs simply because the marginal minority applicant's investment is lower than the marginal non–minority applicant's, and the marginal productivity of investments is diminishing. Since the student body is less diverse, colleges suffer a two dimensional loss of efficiency when affirmative action is prohibited.

The next section describes the setup of the model. Sections 3 and 4 derive the equilibrium results, and section 5 compares the affirmative action equilibrium to the race blind equilibrium. Section 6 and 7 review the theory's implications for human capital investments and the test score gap. Section 8 discusses some commonly proposed alternatives to affirmative action in context of the theory and section 9 concludes.

2 The Model

The model consists of a unit mass of applicants and one college. The timing of the game is as follows. First the college commits to an admissions policy. Then, after observing the admissions policy, applicants make their human capital investment decisions. Finally, test scores are realized and admissions decisions are carried out, with all admitted applicants enrolling.

Before giving the technical description of the model, it is important to point out how minorities and non–minorities differ in the model. I assume that minorities are ex-ante identical to non–minorities in terms of the initial distribution of ability; an assumption supported by Fryer and Levitt (2002) showing insignificant test score differences between black and white children upon entering kindergarten. To differentiate the groups, I assume that minorities are burdened with higher investment cost. This assumption drives many of the results that follow and is necessary since the model is not one in which group differences are based on equilibrium coordination, such as in Coate and Loury (1993). The assumption of differential costs has been used in the economics literature for similar purposes. Fryer et al. (2003) assume that "group 2 is disadvantaged in the sense that it has uniformly less favorable effort cost distribution than group 1" (p. 8), an assumption similar to the one I make below.

Sociologists Fordham and Ogbu (1986) posit that the 'burden of acting white' inhibits blacks' academic performance. Specifically, they argue that blacks' peers discourage them from spending time studying and success in school is viewed as abandoning one's minority identity. That students' peers may tease or ridicule their success in school is an additional social cost of success. Alternatively, psychologists Steele and Aronson (1995) argue the 'stereotype threat' prevents blacks from scoring well on standardized tests when they believe that their performance will be perceived as indicative of their race. To interpret stereotype threat in terms of cost of success, note that it would take more preparation for a black student to score as well as a white student of equal abilities - a higher cost of success.

Finally, the assumption of differential costs is consistent with the central conclusion of Bowen and Bok (1998), that despite lower test scores blacks are worthy of admissions to selective colleges and colleges. In my model, the distribution of innate ability is identical across racial groups, but blacks suffer lower test scores due to investment cost differences. In this way, despite lower test scores blacks are equally deserving of their admission to selective colleges.

2.1 Applicants

The applicants are identified by their innate ability, θ, and their membership to one of two racial groups, B or W. The distribution of ability is identical for each group and is given by the function $F(\cdot)$, with density $f(\cdot)$. Proportion $b < \frac{1}{2}$ of the applicants belong to the minority group B, and the remaining proportion $w = 1 - b$ belong to the majority group W.

Applicants make costly human capital investments $h \in [0, 1]$[6] to prepare for a standardized test which is used by the college for admissions. Test scores are a random function applicants' investments. Let $s(h) = h + \eta$ be the realized test score for investment h, where η is a random disturbance distributed symmetrically over $(-\infty, \infty)$ according to the mean-zero, twice continuously differentiable distribution function $G(\cdot)$:[7]

[6]Investments may take the form of studying, purchase of books, enrollment in college preparatory courses or private school, hiring tutors, etc.

[7]For example, $G(\cdot)$ could be a normal or logistic distribution.

G 1 $G(0) = \frac{1}{2}$ *and* $G(\eta) = 1 - G(-\eta)$,

G 2 $G(\cdot)$ *is twice continuously differentiable,*

G 3 $G''(\eta) > 0$ *for* $\eta < 0$, $G''(\eta) < 0$ *for* $\eta > 0$ *and* $G''(0) = 0$.

Thus, if the college only uses threshold rules (see below), the probability of acceptance for an applicant with investment h and facing threshold t is $p(h|t) = p(s \geq t) = p(h+\eta \geq t) = 1 - G(t-h) = G(h-t)$.

The cost of investment h for type θ is $c(h, \theta)$ for Ws and $\gamma c(h, \theta)$ for Bs, where $\gamma > 1$ by assumption. The cost function $c(h, \theta)$ obeys:

C 1 $c(1, \theta) = \infty$, $\forall \theta$,

C 2 $\lim_{h \to 0} \frac{\partial c(0,\theta)}{\partial h} = 0$, $\lim_{h \to 1} \frac{\partial c(1,\theta)}{\partial h} = \infty$, $\frac{\partial c(h,\theta)}{\partial h} \geq 0$, *and* $\frac{\partial c(h,\theta)}{\partial \theta} \leq 0$ $\forall \theta$, $\forall h$,

C 3 $\frac{\partial^2 c(h,\theta)}{\partial h^2} > 0$, *and* $\frac{\partial^2 c(h,\theta)}{\partial h \partial \theta} < 0$, $\forall \theta$, $\forall h$.

Applicants receive constant benefit A from attending college and are risk neutral with utility that is additively separable in the cost and expected return to investments. Thus for an applicant of innate ability θ, expected utility is defined as a function of investment h:

$$U_j(h, \theta) = A \cdot G(h - t_j) - c_j(h; \theta), \tag{1}$$

where $j \in \{B, W\}$ and $G(h - t_j)$ is the probability of acceptance for chosen investment h and admissions threshold t_j.

Applicants' value to the college is given by their academic ability, which is assumed to be an increasing function of innate ability and investment. Thus in this sense, investments are productive and valued by the college. Let an applicant's academic ability $\hat{\theta}$ be a function of his innate ability θ and investment h, $\hat{\theta} = a(h, \theta)$. Let $a(h, \theta)$ obey:

A 1 $\frac{\partial a(h,\theta)}{\partial \theta} > 0$ *and* $\frac{\partial a(h,\theta)}{\partial h} > 0$, $\forall \theta$, $\forall h$.

A 2 $\frac{\partial^2 a(h,\theta)}{\partial h^2} < 0$ *and* $\frac{\partial^2 a(h,\theta)}{\partial \theta^2} < 0$, $\forall \theta$, $\forall h$.

The academic ability parameter, $\hat{\theta}$, is the parameter of interest to the college, as discussed below.

2.2 The College

The college maximizes the quality of the student body, where quality is defined as an index of the academic ability, $\hat{\theta}$, of the admitted applicants. College quality is an increasing function of applicants' innate abilities and investments. Investments are important to college quality because they increase applicants' academic ability. That is, applicants' investments are productive since $\frac{\partial a(h,\theta)}{\partial h} > 0$.

Applicants' innate abilities, investments and academic abilities are unobserved by the college, so admissions decisions may be conditioned upon test scores and group identity only. To facilitate the analysis of applicants' investment decisions, I restrict attention to threshold admissions policies, admissions policies that admit with certainty all applicants whose scores exceed a given level and reject all applicants whose scores do not. Doing so reduces the college's strategy space to a pair of thresholds $(t_W, t_B) \in ((-\infty, \infty) \times (-\infty, \infty))$.

Let the expected college quality be defined as:

$$Q = Q(T_W(t_W), T_B(t_B)), \tag{2}$$

where $Q(\cdot)$ is increasing, strictly concave and symmetric in the two arguments, and $Q(0, \cdot) = Q(\cdot, 0) = 0$. $T_B(\cdot)$, and $T_W(\cdot)$ are the expected total academic ability of Bs and Ws whose test scores meet threshold t:

$$T_W(t_W) = w \int_0^1 a(h_W^*(t_W, \theta), \theta) f(\theta) G(h_W^*(t_W, \theta) - t_W) d\theta, \tag{3}$$

and

$$T_B(t_B) = b \int_0^1 a(h_B^*(t_B, \theta), \theta) f(\theta) G(h_B^*(t_B, \theta) - t_B) d\theta, \tag{4}$$

where $G(h_j^*(t_j, \theta) - t_j)$ is the probability of acceptance for an applicant of innate ability θ, and $a(h_j^*(t_j, \theta), \theta)$ is the academic ability of an applicant with innate ability θ who chooses investment $h_j^*(t_j, \theta)$.

Finally, the college faces a capacity constraint. The required mass of the accepted applicants (class size) is given exogenously as $S < 1$. Let:

$$M_W(t_W) = w \int_0^1 f(\theta) G(h_W^*(t_W, \theta) - t_W) d\theta, \tag{5}$$

and

$$M_B(t_B) = b\int_0^1 f(\theta)G(h_B^*(t_B,\theta) - t_B)d\theta, \tag{6}$$

be the expected mass of applicants from groups B and W whose test scores are above the appropriate thresholds. Then the capacity constraint is:

$$S = M_W(t_W) + M_B(t_B). \tag{7}$$

3 Applicant Behavior

The game is solvable via backward induction, and therefore I first analyze the applicants' decision problem for any given admissions policy. Let $h^*(t,\theta)$ be the optimal investment for an applicant with innate ability θ who faces threshold t.[8] Then,

$$h^*(t,\theta) \equiv \arg\max_{h\in[0,1]} A\cdot G(h-t) - c(h,\theta). \tag{8}$$

The relevant first order condition is (recall the probability of acceptance for an applicant with investment h is $G(h-t)$):

$$Ag(h^*(t,\theta) - t) = \frac{\partial c(h^*(t,\theta),\theta)}{\partial h}, \tag{9}$$

which states that in equilibrium, the marginal return to investment (LHS) must equal the marginal cost (RHS). Let $A\cdot G(h-t)$ be denoted as the benefit curve for the applicants, and then the first order condition implies equality of the slope of the benefit curve and the slope of the cost curve.[9]

Figure 1 shows the optimal investment of type θ for threshold t. As shown in the figure, the optimal investment is determined where the cost and benefit curves have equal slopes, and the distance between the two curves is minimized. The benefit curve, $A\cdot G(h-t)$, shown in the figure has a point of inflection at $h = t$, and is convex for $h < t$.[10] This type of benefit curve results when the error, η, is distributed normally

[8] The behavioral analysis is generic, but may be applied equally to either group by using the appropriate cost function.

[9] The second order condition is: $Ag'(h^*(t,\theta) - t) \leq \frac{\partial^2 c(h^*(t,\theta),\theta)}{\partial h^2}$.

[10] The point of inflection occurs where $h = t$ because of the symmetry of $G(\cdot)$: $G''(\eta) > 0$ for $\eta < 0$, $G''(\eta) < 0$ for $\eta > 0$ and $G''(0) = 0$. See assumption G 3.

or logistically, for example. Since both the cost curve and benefit curve are convex in this region ($h < t$), multiple equilibria and/or discontinuities may exist in the optimal investment function. These two problems can be avoided by re-scaling either the cost

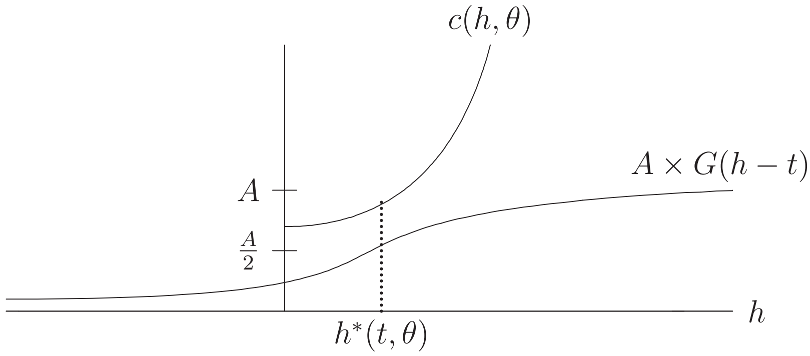

Figure 1: Utility maximization.

curve or the benefit curve, such that the cost curve is 'more convex' than the benefit curve. This is summarized in proposition 3.1:

Proposition 3.1 *There exists an* $A \in (0, \infty)$ *such that for all* $G(\cdot)$, $\theta \in [0, 1]$ *and* $t \in (-\infty, \infty)$, $h^*(t, \theta)$ *exists and is unique. Additionally,* $h^*(t, \theta)$ *is continuous and differentiable in both arguments.*

The existence result is obvious, and the continuity of the best response function derives from the continuity of the cost and benefit curves.

The shape of the optimal investment function $h^*(t, \theta)$ describes the behavior of the applicants. Simplify the notation by suppressing the arguments to the best response and cost functions, $h^*(t, \theta)$ and $c(h, \theta)$. Then, differentiating the first order condition in equation (9) shows that the optimal investment choice is increasing with the applicant's innate ability:

$$\frac{\partial h^*}{\partial \theta} = -\frac{\frac{\partial^2 c}{\partial h \partial \theta}}{\frac{\partial^2 c}{\partial h^2} - Ag'(h^* - t)} > 0, \tag{10}$$

which is greater than zero, given the second order condition in footnote 9 and assumption C 3. Figure 2 illustrates for two applicants with innate abilities θ_1 and θ_2 such that $\theta_2 > \theta_1$. Assumption C 3 guarantees that θ_2, the applicant with higher innate ability, faces a lower marginal cost of investment (flatter cost curve), and thus θ_2 chooses a higher investment than θ_1.

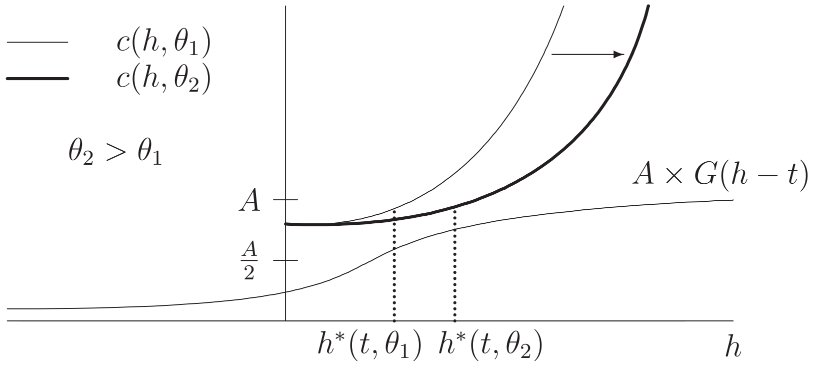

Figure 2: Equilibrium investments are increasing with type.

The following lemma shows how changes in admissions policies affect applicants' investment decisions.

Lemma 3.1 *The optimal investment function, $h^*(t,\theta)$, has the following properties:*

i. If $h^* > t$, then $0 < \frac{\partial h^*}{\partial t} < 1$.
ii. If $h^* < t$, then $\frac{\partial h^*}{\partial t} < 0$.
iii. If $h^* = t$, then $\frac{\partial h^*}{\partial t} = 0$.

Lemma 3.1 states that applicants with high investments (who are applicants with high innate ability), increase their investments when the threshold increases, while applicants with low investments (who are applicants with low innate ability) decrease their investments when the threshold increases. The reason behind this result is that the marginal benefit of investment is decreasing (increasing) with the threshold for applicants with high (low) investments.

Lemma 3.1 is illustrated in figures 3 and 4. Increasing the admissions threshold results in a rightward shift of the benefit curve. In figure 3, the initial choice $h^*(t_1,\theta)$ is greater than the threshold t_1, and thus $h^*(t,\theta)$ rises when the threshold is increased from t_1 to t_2. Figure 4 shows the opposite case.

It is important to point out that part *i.* of lemma 3.1 implies that increasing the admissions threshold leads to decreased probability of acceptance for all applicants,

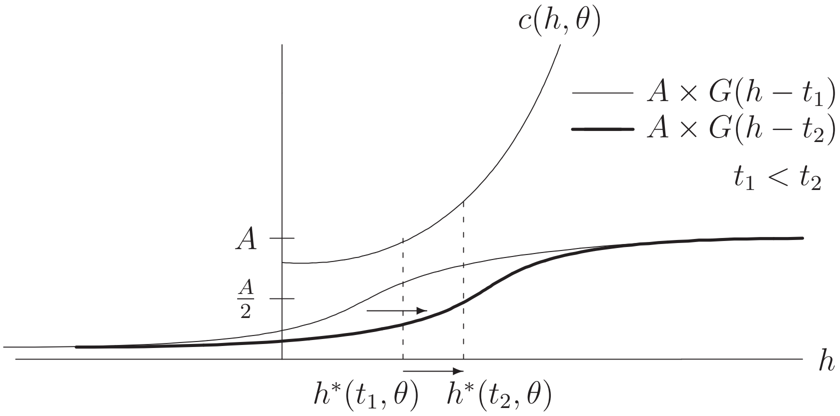

Figure 3: Equilibrium investments are increasing with the admissions threshold.

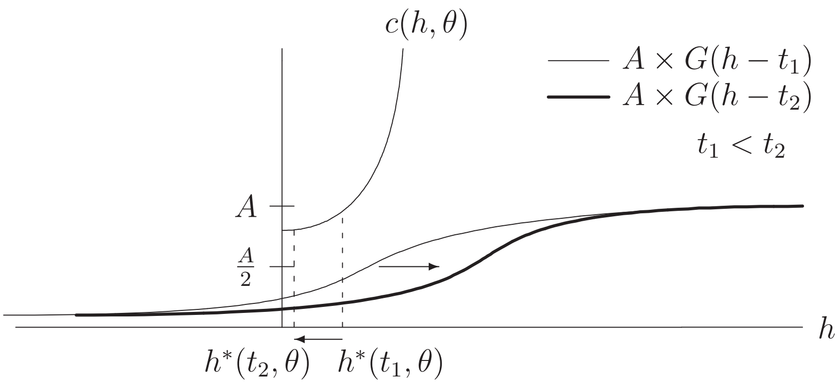

Figure 4: Equilibrium investments are decreasing with the admissions threshold.

regardless of type. Part i. says that that the rate of increase of the investments with respect to thresholds is less than unity. That is, part i. of lemma 3.1 states that $(\frac{\partial h^*}{\partial t} - 1)$ is less than zero for all θ, which implies that change in probability of acceptance, $(\frac{\partial h^*}{\partial t} - 1)g(h^* - t)$, is strictly less than zero.

Note that lemma 3.1 is true for infinitesimal changes in the threshold t. In general, it does not hold for large changes in t. The reason is that there are first and second order effects. For small changes in t, the first order effect of increasing the threshold is dominated by the second order effect, the re-optimization of the applicants. Thus, in the neighborhood of h^*, an applicant choosing an investment above/equal/below the threshold will continue to choose an investment above/equal/below the threshold. For large enough increases in t, clearly all applicants will decrease investments.

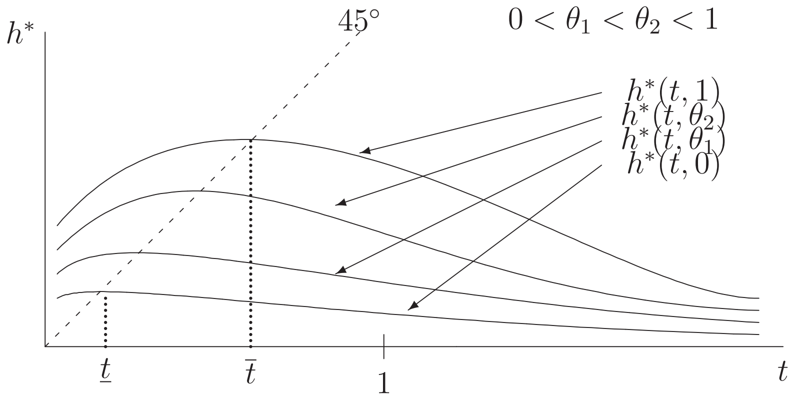

Figure 5: Equilibrium investments as functions of the admissions threshold.

Since the optimal investment functions are continuous, lemma 3.1 gives a complete description of applicant behavior. Figure 5 shows optimal investments functions, $h^*(t, \theta)$, for four applicants with different innate abilities. For a given threshold, applicants with higher innate ability choose higher investments; therefore the optimal investment function for θ_2 lies everywhere above the optimal investment function for θ_1. If a particular applicant chooses an investment above the threshold, increasing the threshold leads her to choose a larger investment, up to a point. Eventually, increasing the threshold will cause her to decrease her investment.

One note is necessary to show that the behavior illustrated in figure 5 is correct. When $t = h = 0$, the marginal benefit of investment is greater than the marginal cost ($\lim_{h \to 0} \frac{\partial c(0,\cdot)}{\partial h} = 0$ and $g'(0 - t) > 0$) and when $t = h = 1$, the marginal cost exceeds the marginal benefit ($\lim_{h \to 1} \frac{\partial c(1,\cdot)}{\partial h} = \infty$ and $g'(0 - t)$ is finite) for all types θ. This means that for $t = 0$, all applicants choose investments above the threshold ($h^* > t = 0$) and for $t = 1$, all applicants choose investments below the threshold ($h^* < t = 1$). Since $h^*(t, \theta)$ is continuous in t, lemma 3.1 implies that applicant behavior is as depicted in figure 5.[11]

Figure 5 displays the representation of applicant behavior described in section 1

[11] Additionally, I should comment on the second derivative of the applicants best response function $h^*(t, \theta)$ with respect to t. Initially the sign is negative as investments increase at a decreasing rate when thresholds increase. Eventually, investments decrease with increases in thresholds, but investment reach an asymptote at zero for all θ.

and alluded to by Justice Thomas in his *Grutter* opinion. For admissions thresholds that are not extremely high or low, it should be expected that changing the threshold will cause some applicants to increase investments and some applicants to decrease investments. These countervailing effects complicate the task of evaluating the effects of policy changes. Before addressing these types of questions, it is necessary to determine precisely how colleges change admissions policies in response to a ban on affirmative action.

4 College Behavior and Affirmative Action Equilibrium

When the college is allowed to use affirmative action in its admissions policy, it may select different admissions rules for Bs and Ws. Conversely, when affirmative action is prohibited, the college must use the same admissions criterion for both groups. Let the affirmative action equilibrium be denoted as an optimal policy pair, (t_W^*, t_B^*), and an equilibrium without affirmative action as $t_W^{NoAA} = t_B^{NoAA} = t^{NoAA}$. Note that there is no ex ante requirement that $t_W^* > t_B^*$ in the affirmative action equilibrium. The college may choose any pair of thresholds when affirmative action is permitted.

Formally, the college chooses (t_W, t_B) to maximize $Q(T_W(t_W), T_B(t_B))$ subject to $S = M_W(t_W) + M_B(t_B)$. Forming the Lagrangian $L(t_W, t_B, \lambda)$, (t_W^*, t_B^*) are defined by:

$$(t_W^*, t_B^*) \equiv \arg \max_{(t_W, t_B, \lambda)} Q(T_W(t_W), T_B(t_B)) + \lambda(S - M_W(t_W) - M_B(t_B)). \tag{11}$$

Before stating the existence result, some additional notation is necessary. Let $\tilde{t}_W$ and $\tilde{t}_B$ be defined by $S = M_W(\tilde{t}_W) + M_B(\infty) = M_W(\infty) + M_B(\tilde{t}_B)$. Note that $\tilde{t}_B$ exists only if there sufficient applicants from group B to fill the entire class, i.e. if $b > S$. To simplify the analysis, I adopt this assumption for the remainder of this paper.[12]

[12] On the surface, this assumption appears invalid. For example in the fall of 2002, $11,719$ first time freshmen were admitted to UT–Austin, but only 1,080 blacks applied for admissions. See http://www.utexas.edu/academic/oir/statistical_handbook/02-03/students/s25/. However, if UT–Austin administrators desired an all black student body, they might admit

Proposition 4.1 *There exists at least one affirmative action equilibrium, denoted* (t_W^*, t_B^*), *such that* t_W^* *and* t_B^* *are finite, and* $\tilde{t}_W < t_W^*$ *and* $\tilde{t}_B < t_B^*$.

Note that the set of feasible thresholds is characterized by $t_W^* \in [\tilde{t}_W, \infty)$ and $t_B^* \in [\tilde{t}_B, \infty)$, so the proposition guarantees existence of an equilibrium on the interior of the feasible set. The existence result is not difficult to obtain, given the concavity assumptions on $Q(\cdot)$. However, the proof is not instructive, so I give a more intuitive analysis below.

Combine the two first order conditions from the maximization problem (11) and eliminate the multiplier, λ, to obtain the first order optimality condition:

$$-\frac{\frac{\partial Q(T_W(t_W^*),T_B(t_B^*))}{\partial T_B}T_B'(t_B^*)}{\frac{\partial Q(T_W(t_W^*),T_B(t_B^*))}{\partial T_W}T_W'(t_W^*)} = -\frac{M_B'(t_B^*)}{M_W'(t_W^*)}. \tag{12}$$

To gain some intuition for equation (12), consider the level curves of the college's constraint and objective functions. Define the *isosize* curve as all possible combinations of thresholds (t_W,t_B) which satisfy the capacity constraint, and the *isoquality* curve as the set of thresholds (t_W,t_B) which yield the same college quality.

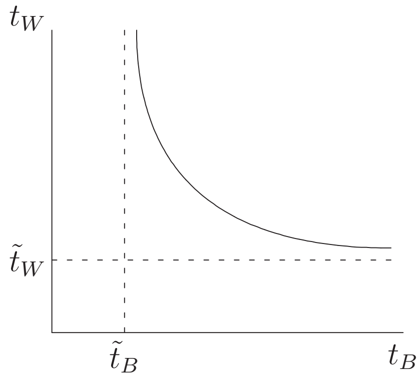

Figure 6: The isosize curve of a typical college.

Figure 6 shows a well behaved isosize curve. The right hand side of equation (12) gives the slope of the isosize curve. Since the $M_j'(\cdot)$ are less than zero for all t (see

all black high school graduates. This admissions policy would more than fill the freshman class, since 28,295 black students graduated from Texas high schools in 2001. See http://www.utexas.edu/student/research/reports/admissions/Pipeline2001.pdf. Thus the feasibility of admitting a class composed entirely of minority candidates should not be questioned.

lemma 10.1 in the appendix), the slope of the isosize is strictly less than zero.[13] That is, an increase in t_B must be accompanied by a decrease in t_W in order to hold constant the mass of the admitted applicants. Note that $\tilde{t}_W$ and $\tilde{t}_B$ give the boundaries of the isosize curve (for $S < 1$), so that the isosize curve must asymptote vertically at $t_B = \tilde{t}_B$ and horizontally at $t_W = \tilde{t}_W$. Additionally, higher isosize curves correspond to smaller colleges.

The isosize is not guaranteed to be convex everywhere for all parameterizations, but in most cases convexity does hold. Thus figure 6 depicts a well behaved but typical isosize. The convexity of the isosize curve reflects the fact that the $M(\cdot)$ are less responsive to changes in high thresholds than low thresholds. For example, if t_B is very high, there are few B's admitted and their investments are insensitive to changes in t_B, so decreasing t_B will have minimal impact on $M_B(t_B)$. Conversely, if t_W is low many W's are being admitted and increasing t_W by the same amount will result in relatively larger change in $M_W(t_W)$.

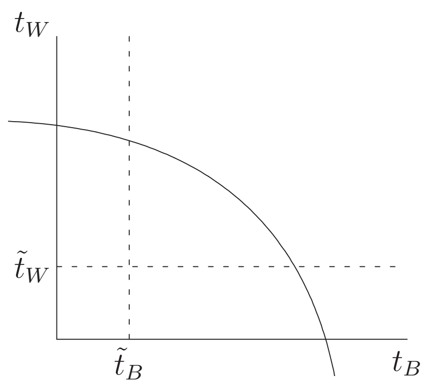

Figure 7: The isoquality curve of a typical college.

Figure 7 depicts a typical isoquality curve. The left hand side of equation (12) gives the slope of the isoquality curve. By assumption, $\frac{\partial Q}{\partial T_W} > 0$, and $\frac{\partial Q}{\partial T_B} > 0$, and see lemma 10.2 in the appendix for the result that $T'(\cdot) < 0$.[14] Therefore, the isoquality

[13] It is a trivial result that the mass of applicants accepted from a given group is decreasing with the threshold.

[14] Since $T_j(\cdot)$ measures the academic ability of the accepted applicants, it is not clear that $T_j'(\cdot)$ is decreasing. A problem arises when the threshold is low; an increase in a low threshold brings an increase

curve is downward sloping. Intuitively, an increase in t_W leads to decrease in $T_W(\cdot)$ and thus a decrease in $Q(T_W(t_W), T_B(t_B))$. To hold the quality of the admitted applicants constant, t_B must increase. Note that thresholds may be negative and the isoquality curve is defined on the region $(t_W, t_B) \in (-\infty, \infty) \times (-\infty, \infty)$. Also, lower isoquality curves correspond to higher quality.

Similar to the description of the isosize, I have not commented on the curvature of the isoquality curve. In general, there are regions of convexity and concavity on the isoquality curve, but the overall shape is concave. When t_B is low, the college would agree to a significant raise in t_B in exchange for a small decrease in t_W. That is, when Bs are relatively plentiful on campus, the college is willing to trade a large number of admits from group B to gain a small number of admits from group W, holding quality constant. An assumption of sufficient concavity in the quality function $Q(\cdot)$, while helpful for the illustrations shown but not necessary for equilibrium, would guarantee a region of concavity on the isoquality where t_W and t_B are greater than $\tilde{t}_W$ and $\tilde{t}_B$, respectively, as shown in figure 7.

Proposition 4.1 is illustrated in figure 8. Very simply, the equilibrium thresholds (t_W^*, t_B^*) are determined by the point of tangency between the isosize and isoquality curves. Proposition 4.1 does not guarantee uniqueness and does not require any additional assumptions on $Q(\cdot)$, other than those stated in section 2. However, if the isosize is convex and the isoquality is concave on $(t_W, t_B) \in [\tilde{t}_W, \infty) \times [\tilde{t}_B, \infty)$, then the equilibrium is unique, as shown in the figure.

Proposition 4.2 *If the quality function is sufficiently concave, then the equilibrium thresholds obey $t_W^* > t_B^*$. That is, when affirmative action is permitted, the college gives preferential treatment to the minority.*

---

in investment for many applicants, which may outweigh the decline in the mass of applicants accepted. If the threshold is large enough, the condition in lemma 10.2, $\frac{\frac{\partial h_j^*}{\partial t_j}}{\frac{\partial h_j^*}{\partial t_j} - 1} > -\frac{a(h_j^*, \theta)}{\frac{\partial a(h_j^*, \theta)}{\partial h}} \frac{g(h_j^* - t)}{G(h_j^* - t)}$ for all t and θ, is met trivially since $\frac{\partial h^*}{\partial t} < 0$, $\frac{\partial h^*}{\partial t} - 1 < 0$, and the right hand side is less than zero.Additionally, the lemma will be satisfied if the academic abilities are large relative to the marginal changes in academic abilities (i.e. $a(h, \theta)$ is large relative to $\frac{\partial a}{\partial h}$) or if the marginal change in probability of acceptance is large relative to the probability of acceptance (i.e. $g(h^* - t)$ is large relative to $G(h^* - t)$, which is true for high thresholds). Thus I adopt the condition of lemma 10.2 so that $T_j'(\cdot) < 0$.

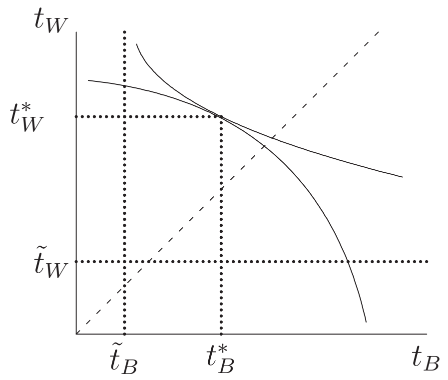

Figure 8: College equilibrium.

Proposition 4.2 is conditioned upon sufficient concavity of the college quality function. However, this condition is not necessary and in general it is not very restrictive. The proof of proposition 4.2 is by contradiction, and in the appendix, I show that if $t_W^* < t_B^*$, then the first order condition in equation (12) is not satisfied. First, re-state the first order condition as:

$$\frac{\frac{\partial Q(T_W,T_B)}{\partial T_B}}{\frac{\partial Q(T_W,T_B)}{\partial T_W}} = \frac{T'_W(t_W^*)}{T'_B(t_B^*)} \frac{M'_B(t_B^*)}{M'_W(t_W^*)} \tag{13}$$

If the quality function is concave in each argument and $t_W^* < t_B^*$, it must be true that $\frac{\partial Q(T_W,T_B)}{\partial T_B} > \frac{\partial Q(T_W,T_B)}{\partial T_W}$, with the inequality becoming stronger for more concave quality functions. Since the ratio $\frac{T'_W(t_W^*)}{T'_B(t_B^*)} \frac{M'_B(t_B^*)}{M'_W(t_W^*)}$ is bounded, a contradiction is obtained for sufficiently concave $Q(\cdot)$. Additionally, since the ratio $\frac{T'_W(t_W^*)}{T'_B(t_B^*)} \frac{M'_B(t_B^*)}{M'_W(t_W^*)}$ is likely close to 1, the restriction imposed on the quality function is not stringent, and is likely satisfied by minimal concavity of $Q(\cdot)$.[15]

To understand the intuition behind this result, suppose first that there is a common threshold, t^*, and recall that the applicants' investments are productive. Since Ws are more likely to chose an investment above $t_W^* = t^*$, increasing t_W^* leads relatively more of them to increase investments, compared to increasing t_B^* for Bs. Similarly, since Bs are relatively more likely to choose an investment below $t_B^* = t^*$, decreasing their threshold leads relatively more of them to increase investments, compared to decreasing t_W^* for

[15]This is true because the $T(\cdot)$ and $M(\cdot)$ functions are defined similarly and therefore, $T'_W(t_W^*) \approx M'_W(t_W^*)$ and $T'_B(t_B^*) \approx M'_B(t_B^*)$.

Ws. This is complimented by the fact that under a common threshold, the expected innate ability of the marginally admitted B is greater than the expected innate ability of the marginally admitted W, so that investments are more productive for the marginal B than the marginal W. Thus, it should be optimal for the college to set a higher threshold for Ws than for the Bs.

5 Comparing Admissions Regimes

Banning affirmative action requires that the same admissions rule is used for both groups. Thus $t_B^{NoAA} = t_W^{NoAA} \equiv t^{NoAA}$. Therefore, the capacity constraint given in (7) characterizes the equilibrium. The preceding analysis showing that the isosize curve is strictly decreasing guarantees that t^{NoAA} is unique. To find t^{NoAA}, simply pick the point where the 45–degree line crosses the isosize curve. Comparing the quality of the admitted applicants under the two different admissions regimes gives the following result:

Proposition 5.1 *The expected quality of the class of accepted students decreases when affirmative action is banned. That is,* $Q(T_W(t_W^*), T_B(t_B^*)) > Q(T_W(t^{NoAA}), T_B(t^{NoAA}))$*. The college prefers to use affirmative action.*

Proposition 5.1 is illustrated in figure 9. Since the affirmative action equilibrium lies above the 45 degree line (see figure 8), it is a straightforward conclusion that the the isosize cuts the isoquality from below when affirmative action is banned. Since the two curves are not tangent, it must be that restricting the college to race blind admissions policies decreases the quality of the admitted students. The college would prefer to admit more B's and fewer W's by lowering t_B and raising t_W, giving a revealed preference proof of proposition 5.1.

Affirmative action allows the college to increase the quality of the accepted students. As discussed above, when affirmative action is permitted, the college is able to take advantage of the fact that investments are more productive for the marginally admitted B than the marginally admitted W. Thus affirmative action allows the college to increase the productivity human capital investments, and increase college quality.

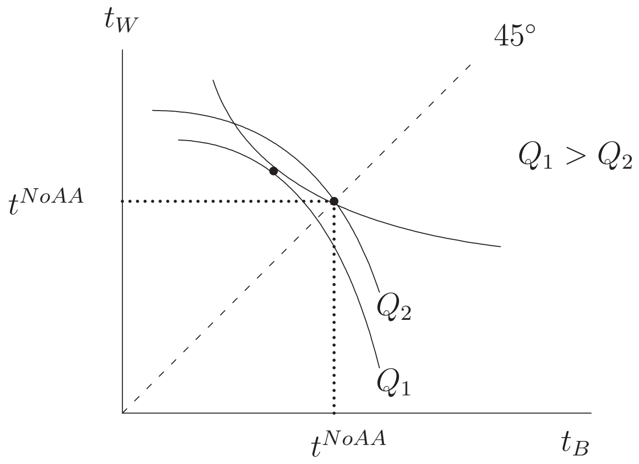

Figure 9: Affirmative action is preferred by the college.

To conclude the comparison of the equilibria with and without affirmative action, compare group representation before and after affirmative action is banned:

Proposition 5.2 *If $t_W^* > t_B^*$ minority representation decreases when affirmative action is banned.*

This is a simple result based on the fact that the mass of applicants from one group is decreasing with the admissions threshold.

Together, propositions 4.2, 5.1 and 5.2 confirm current opinions and data about the use of preferential admissions and bans on such policies, validating the model. Proposition 4.2 shows that when affirmative action is permitted the college gives preferential treatment to the minority, widely consistent with collegiate admissions policies. For example, the University of Michigan used a point system (although now banned by The Supreme Court) which gave 20 bonus points to underrepresented minority applicants, thereby permitting admissions with lower test scores. Proposition 5.1 shows that when given the choice, colleges prefer to use affirmative action and that affirmative action maximizes the quality of the student body. Revealed preference confirms that colleges prefer to use preferential admissions policies and defer to Bowen and Bok (1998) and Gurin (2000) to show that affirmative action maximizes the educational benefits accruing to all students. Finally, proposition 5.2, which shows that minority enrollment decreases when affirmative action is banned, confirms known data and simulations (refer to the statistics in the introduction or see Bucks (2002), Epple et al. (2003), Kane (1998),or Long (2002b)).

6 The Distribution of Investments and Social Welfare

The preceding sections studied the equilibria of the model when affirmative action is available as an admissions policy and when it is not. In the discussion that follows, I study the effects of banning affirmative action on the distribution of human capital investments within each racial group. The main challenge, as suggested by lemma 3.1, is to evaluate the consequences of policy changes when applicants within each group react differently to changes in admissions policy. Focusing on the distributional consequences allows me to weigh the costs (some decreases in investment) and the benefits (some increases in investment) of prohibiting affirmative action.

The results that follow allow me to compare admissions regimes in terms of social welfare of each racial group of applicants. Unfortunately, only in trivial cases are global social welfare judgements about the effects of a policy change possible and further discussion of these cases is omitted. Unless otherwise noted, the discussion and results below apply to the social benefits of a policy change for either group, B or W.

Recall that the investments of the applicants are productive. Investment makes applicants more valuable to the college, and represents an increase in their stock of human capital. Even though the applicants themselves do not value the investments and their resulting academic ability, human capital investment is undeniably valuable to a racial group as a whole. Thus, a cumulative investment distribution that is to the right of another should be preferred socially – the distribution to the right entails higher investments for all applicants. The primary indicator of welfare used in this analysis is the applicants' investments.

Notice that the definition of first order stochastic dominance (FOSD) is satisfied if a policy change causes all applicants in a group to increase or decrease their investments.[16] Thus in the case at hand, the most desirable investment distribution is the one that first order dominates all others. Unfortunately, when comparing the effects of changing admissions thresholds rarely does a first order dominant distribution emerge. As implied by figure 5 in section 3 and shown below in figure 10, in many cases FOSD is unavailable for comparing distributions because the cumulative distribution functions of investment resulting from any two thresholds, t_1 and t_2, cross one another. Only when the changes in investment are uniform in direction will FOSD be useful as a criteria for comparing admissions policies.

To see that distributions may cross, recall Lemma 3.1. Let $\underline{t}$ be defined by $\frac{\partial h^*(\underline{t},\underline{\theta})}{\partial t} = 0$, and $\bar{t}$ be defined by $\frac{\partial h^*(\bar{t},\bar{\theta})}{\partial t} = 0$.[17] Thus for thresholds less than $\underline{t}$, optimal investments are increasing with the threshold for all θ, and for thresholds greater than $\bar{t}$, investments are decreasing with the threshold for all θ. For thresholds between $\underline{t}$ and $\bar{t}$, investment changes are positive for some applicants and negative for others when the threshold increases. These three ranges cover the space of possible thresholds (see figure 5).

Thus distributions may cross when $(t_1, t_2) \in (\underline{t}, \bar{t}) \times (\underline{t}, \bar{t})$, eliminating first order dominance as a criteria for ranking thresholds in that region. With some abuse of notation, denote the distribution of investments as $F(h|t) = F(\theta(h,t))$.[18] Lemma 6.1 gives the formal result that distributions cross.

Lemma 6.1 *For all t_1, $t_2 \in (\underline{t}, \bar{t})$, $\theta \in [0,1]$, $F(h|t_1)$ and $F(h|t_2)$ cross at least once. Additionally, if t_1, t_2 satisfy $\frac{\partial^3 c}{\partial h^3} = 0$, $\frac{\partial^3 c}{\partial h^2 \partial \theta} = 0$, $g''(h^*(\bar{t},\underline{\theta}) - \bar{t}) < 0$ and $g''(h^*(\underline{t},\bar{\theta}) - \underline{t}) < 0$ for all θ, then $F(h|t_1)$ and $F(h|t_2)$ cross exactly once.*

Lemma 6.1 is illustrated in figure 10. If investment distributions cross at least once, neither distribution will be first order dominant. The second part of the lemma, that investment distributions cross exactly once, is not necessary to eliminate FOSD as

[16] Distribution $F(h|t_1)$ FOSD distribution $F(h|t_2)$ if for all h, $F(h|t_1) < F(h|t_2)$.

[17] Equivalently, $h^*(\bar{t},\bar{\theta}) = \bar{t}$ and $h^*(\underline{t},\underline{\theta}) = \underline{t}$.

[18] The cumulative distribution of investments is given by the probability that the chosen investment is less than a given level, $\hat{h}$. $P(h \leq \hat{h}) = P(\theta \leq \theta(\hat{h},t)) = F(\theta(\hat{h},t))$. Therefore, to find the distribution of investments, use the distribution of types evaluated at the type who chooses investment $\hat{h}$.

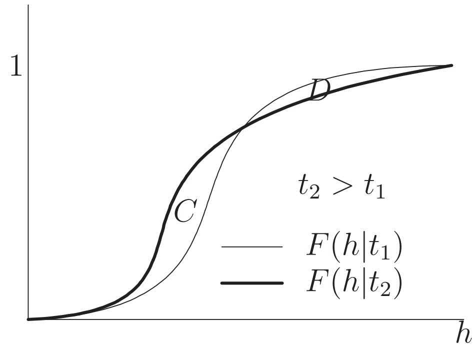

Figure 10: Investment distributions cross when threshold increases.

a comparison criterion, but is used below to establish second order stochastic dominance (SOSD). The conditions stated in the second part of lemma 6.1 are not necessary and are technical in origin.[19]

Since the interesting as well as the more likely cases are those where the direction of change in applicant behavior is not uniform in response to changes in admissions policies, it is natural to ask if second order stochastic dominance (SOSD) can be used to rank admissions policies when FOSD relationships are absent.[20] Using second order dominance as a criteria to compare distributions is not without precedent in the economics literature. Risky assets which are characterized by probability distributions have long been compared with second order dominance, see Levy (1992) for a thorough survey.

Proposition 6.1 gives the formal statement of the stochastic dominance results:

Proposition 6.1 *Suppose* $t_1 > t_2$.

i. For all t_1, t_2 *satisfying* $t_1 \in (-\infty, \underline{t}]$, $t_2 \in (-\infty, \underline{t})$, $F(h|t_1)$ *FOSD* $F(h|t_2)$.

[19]The first two conditions (third derivatives of the cost function) merely simplify the proof, but the final two conditions do have relevance. Since the discussion pertains to thresholds which are neither extremely high or low (thresholds in the range $(\underline{t}, \bar{t})$), it is reasonable to conclude that all applicants choose investments which are neither significantly higher or lower than either threshold. If investments are "close" to the threshold, then $|h^* - t|$ is small and $g(h^* - t)$ will fall near the center of mass of the distribution $G(\cdot)$. Since assumption G 3 is assumed to hold, in which case $G(\cdot)$ resembles a standard normal distribution, for example, then it is likely that $g''(h^* - t)$ is less than zero for all θ, as stated in the lemma.

[20]Distribution $F(h|t_1)$ SOSD distribution $F(h|t_2)$ if for all h, $\int_{\underline{h}}^{h} F(x|t_1)dx < \int_{\underline{h}}^{h} F(x|t_2)dx$.

ii. *For all* t_1, t_2 *satisfying* $t_1 \in (\bar{t}, \infty)$, $t_2 \in [\bar{t}, \infty)$, $F(h|t_2)$ *FOSD* $F(h|t_1)$.

iii. *There exists* $\hat{t}$ *such that for all* t_1, t_2 *satisfying* $t_1 \in (\hat{t}, \bar{t}]$, $t_2 \in [\hat{t}, \bar{t})$, *and if distributions* $F(h|t_2)$ *and* $F(h|t_1)$ *cross exactly once,* $F(h|t_2)$ *SOSD* $F(h|t_1)$.

Note that first order dominance implies second order dominance. The implications of SOSD discussed below apply equally to cases *i.* and *ii.* of proposition 6.1, where FOSD is applicable. Therefore I will not directly discuss FOSD any further.

Proposition 6.1 states that for selective colleges, there is a strict ranking of thresholds based on SOSD. For all thresholds above $\hat{t}$, the dominant investment distribution is the one induced by the lower threshold. To see the intuition behind proposition 6.1 consider small decreases in relatively high thresholds. The small decrease will cause a small group of high ability applicants to decrease investments and a large group of mid and low ability applicants to increase investments. Examining figure 10, this causes area C to be larger than area D.

Thus, for colleges who are sufficiently selective, a higher admissions threshold will result in a dominated investment distribution. In technical terms, a selective college is one that chooses thresholds higher than $\hat{t}_W$ and $\hat{t}_B$ for the two groups respectively. Indeed, selective colleges are most likely to employ affirmative action admissions policies, and are appropriately the focus of my analysis. Empirical evidence that affirmative action is primarily practiced by selective colleges can be found in Kane (1998) and Long (2002b). Using the High School and Beyond data, Kane shows that the effect of being black or Hispanic on an applicant's predicted probabilities of acceptance increases with selectivity. Specifically, conditioning on a wide set of covariates, Kane estimates that compared to whites, blacks are 2.0 percent less likely to be admitted to colleges in the third quartile of selectivity, but 13 percent more likely to be admitted to colleges in the top quartile of selectivity. Long shows a similar effect using SAT data obtained from the College Board: "the preference given underrepresented minorities increases as the college's median freshman test score increases and is positive for colleges whose median freshman scores above 889." (p. 11) Although Long's results show that underrepresented minorities are given positive preference at 73 percent of four year colleges, the strength of the preference is increasing with selectivity.

When comparing two investment distributions, the admissions threshold which yields the second order stochastically dominant distribution maximizes the group's average investment. This result is well know in the stochastic dominance literature and is simple to prove. Applying this result to proposition 6.1 shows that thresholds can be ranked by mean investment. Intuitively, for high thresholds many applicants choose investments which are below the threshold (see figure 5). Then, decreasing the threshold will cause most applicants to raise investments, thereby increasing the average. Eventually, continued decreases of the threshold will result in more applicants decreasing their investments, and the average will fall.

The following result compares the aggregate investments of each group when affirmative action is permitted and when it is not.

Proposition 6.2 *For a selective college, if $t^*_W > t^*_B$, then affirmative action increases the average group investment for B and decreases average group investment for W. That is, $E(h_B|t^{NoAA}) < E(h_B|t^*_B)$, and $E(h_W|t^{NoAA}) > E(h_W|t^*_W)$.*

Proposition 4.2 established that Bs (Ws) face a higher (lower) threshold when affirmative action is banned, and combined with proposition 6.1 this gives the result that for Bs (Ws), the investment distribution under affirmative action dominates (is dominated by) the investment distribution under race blind admissions. The preceding paragraph gives the implications for the average investment levels, that when affirmative action is banned, the average investment will increase for Ws and decrease for Bs. Since investments indicate the strength of incentives, affirmative action maximizes aggregate incentive for human capital investment for Bs. See figure 11 for an illustration of proposition 6.2.

Proposition 6.2 does inform us about the changes in average investment of a group, but it does not make any statements about social welfare. For social welfare functions linear in applicants investments and giving equal weight to all applicants' investments, proposition 6.2 does have welfare implications: lower average group investment corresponds to lower social welfare. The following result shows how to rank different admissions policies in terms of the social welfare of each racial group, for a general specification of social welfare.

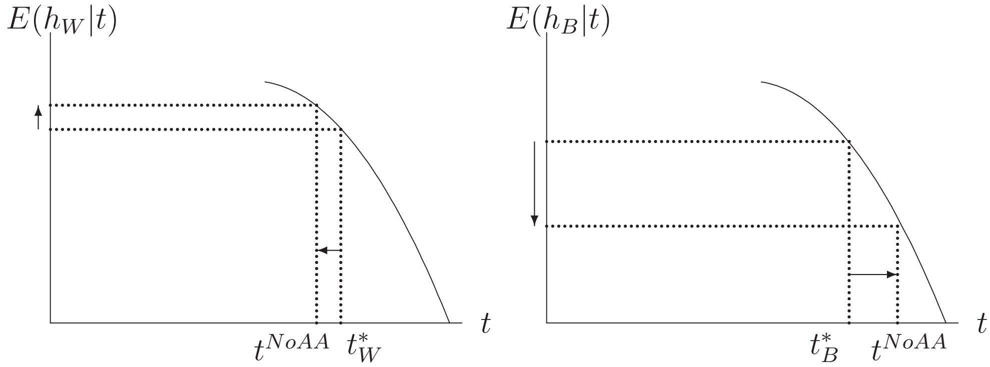

Figure 11: Affirmative action increases Bs' average group investment and decreases Ws' average group investment.

Proposition 6.3 *Let social welfare of a racial group be defined as* $U = \int u(h)dF(h|t)$ *where* $u(\cdot)$ *is increasing and strictly concave. For a selective college, if* $t_W^* > t_B^*$*, then affirmative action increases Bs' social welfare and decreases Ws' social welfare.*

Proposition 6.3 simplifies the interpretation of second order dominance of investment distributions - the dominant distribution corresponds to higher social welfare. Note that affirmative action, which is characterized by a lower admissions threshold for Bs compared to the race blind case, increases Bs' social welfare. By lowering the admissions threshold, the higher investments of the low and middle ability students outstrip the lower investments of the high ability students.

An alternative interpretation of proposition 6.3 is available when the social welfare function is viewed as the expected utility of an applicant before her type, θ, has been chosen by nature. If this applicant is risk averse over the realization of her type, and hence risk averse over the possible optimal investments that she will choose, then her expected utility is as given in the proposition. Another interpretation is available from the perspective of a social planner. If the social planner is risk averse over the randomly drawn applicant's investment, then the expression for social welfare in proposition 6.3 gives the expected utility of the randomly drawn applicant.

The effects of banning affirmative action on Bs are amplified if the social welfare calculation includes the benefits of college education. Recall that the probability of admissions $p(s > t) = G(h^*(t, \theta) - t)$ is decreasing in t for all types θ (see lemma

3.1). Thus including the benefits of college, A, in the specification of social utility, strengthens the effects of a ban. Since all Bs have lower probability of attending college when affirmative action is banned, welfare is further reduced. This analysis can be taken one step further considering that affirmative action maximizes college quality. Thus the benefits of college are less when affirmative action is banned, further compounding the consequences of a ban for Bs.

Ws are better off under race-blind admissions than affirmative action. Proposition 6.3 shows that when affirmative action is banned, the social utility of Ws increases. However, there are some mitigating factors. First, as shown by Kane (1998), the changes in whites' probabilities of acceptance to selective colleges resulting from bans in affirmative action are very small. Due to the fact that whites outnumber blacks in the population by a factor of roughly ten to one, the admission of an additional B has a small effect on Ws' chances of admissions. Second, as discussed above the benefits of attending college for those Ws who are admitted is greater under affirmative action. Even though fewer Ws attend college when affirmative action is used, those who do attend receive greater benefits.

7 Distribution of Test Scores

The previous section discusses the distributional effects of banning affirmative action on applicants' investments. Since investments are not directly observable, these results are not empirically verifiable. However, recall that applicants' test scores are a random function of their investments, thus determining the relationship between test score distributions and the underlying investment distributions is very desirable.

Proposition 7.1 *For a selective college, if $t^*_W > t^*_B$, the average test score of Bs (Ws) decreases (increases) when affirmative action is banned. Banning affirmative action increases the black-white test score gap.*

Proposition 7.1 is a direct implication of 6.2, which states that affirmative action increases Bs' average investment and decreases Ws' average investment. Since $s = h + \eta$, the average test score equals the average investment.

Changes in the test score gap do not provide any information about the welfare of either group. Unless social welfare is a monotonic function of the average investment, proposition 7.1 has no social welfare implications. Comparing investment and test score distributions allows the analyst to use proposition 6.3 to make welfare inferences based upon observed test scores. Let $H(\cdot|t)$ be the distribution of test scores for a given threshold t, within a racial group.

Proposition 7.2 *In the sense of second order stochastic dominance,* $H(s|t_1)$ *second order dominates* $H(s|t_2)$ *if and only if* $F(h|t_1)$ *dominates* $F(h|t_2)$*.*

Proposition 7.2 implies that social ranking of admissions policies based on test score distributions is equivalent to rankings based on investment distributions. This result allows social welfare and human capital investment inferences using observed test score distributions.

8 Policy Alternatives

As hinted by Justice O'Connor in the Opinion of the Court in *Grutter*, race conscious admissions policies are likely have a finite lifespan in the United States. Her suggestion that affirmative action policies will ultimately be outlawed within 25 years of the *Grutter* decision begs the analyst to ask whether race conscious policies will be needed in 25 years. In terms of the model, as long as investment cost differentials persist, the test score gap will persist, and will be exacerbated by limiting preferential admissions.

Administrators and educators usually have a two pronged strategy to deal with the problems evidenced by and brought about by the test score gap.[21] First, many programs are designed to increase minorities' school achievement and thereby increase minorities' test performance and college enrollment.[22] This policy is interpreted in terms of the

[21] Note that analysis of Top-X percent programs, which have been implemented in Texas and Florida is not feasible in the model. The reason is that admissions with Top-X percent programs is governed by high school grades, which may vary over different high schools for applicants with the same innate ability. Additionally, such analysis requires assumptions about the distribution of innate ability across high schools.

[22] For example California's Early Academic Outreach Program, http://www.eaop.org/.

model as attempting to reduce the investment costs for Bs. Recall that the cost of investment for Ws is $c(h, \theta)$ and the cost of investment for Bs is $\gamma c(h, \theta)$, where $\gamma > 1$. Thus any policy that seeks to limit the differences in cost of investment operates by decreasing γ.

Suppose that affirmative action is not permitted, and consider a decrease of γ. Bs face lower marginal cost of investment, and thus all Bs should increase their investments. However, the increased investments of the Bs means that more Bs are going to meet the college's admissions threshold, so the college must increase the threshold for all applicants in order to maintain the size of the student body. Increasing the threshold means that fewer Ws are admitted and therefore Bs' representation at the college increases. Note that this policy must increase college quality, since the student body is more diverse and investments are more productive.

The effect of this policy on Ws' investments is clear. Using propositions 6.2 and 6.3, we know that Ws' aggregate investment and social welfare decreases, but again the effect is tempered because the benefits to college attendance are greater. The effect of this policy on Bs' investments is not so clear. There are two countervailing influences on Bs' investments – lower marginal investment cost and higher threshold. In terms of the human capital investments, Bs with high innate ability respond to both of these influences with higher investments. While decreased costs leads low and mid ability Bs to increase their investments, an increased admissions threshold causes them to decrease their investments. The overall effect on low and mid ability Bs is ambiguous, and thus the implications for aggregate human capital investments are also ambiguous. It is clear that this policy will increase inequality within the population of Bs, both in terms of human capital investments and probabilities of admission to college. As a group, Bs benefit from greater enrollment and greater benefits of college attendance, but the change in aggregate investment is ambiguous.

Second, scholarship programs and minority recruitment programs seek to increase minority college enrollment by increasing the expected or perceived net benefits of college attendance.[23] Suppose that the utility function for group j is specified as

[23] Note that I characterize scholarship programs as increasing the net benefits of college attendance.

$U_j(p_j(h), h; \theta) = p_j(h|t) \cdot A_j - c_j(h, \theta)$, where $j \in \{B, W\}$. This specification differs from the specification in equation (1) because A is allowed to vary between the two groups. Many such policies are designed to increase the net payoff of college to minority students. Minority scholarships and equal opportunity laws in the workplace all have the effect of increasing the net payoff to college for minority students.[24]

This policy is analytically similar to the cost intervention policy; it seeks to increase the marginal return to investment. As above, this will soften the need for affirmative action, increasing college quality. In terms of investments and rates of college attendance, the effects are qualitatively the same as the cost intervention policy. Again, the change in aggregate investment resulting from this policy are ambiguous because low and middle ability Bs may or may not increase investment. The differences is that if the benefits of college are considered, this policy benefits the high ability B's more than the low ability Bs when compared to the cost reduction policy because the benefits of a scholarship program are only enjoyed by those applicants who are admitted. The difference in probability of acceptance between high and low ability B's widens as above, but now the increased benefits of college are only enjoyed by the high ability Bs.

9 Conclusion

Recently, The Supreme Court of the United States upheld the use of race conscious admissions policies, but individual states continue to ban affirmative action practices through legislation and voter referenda. Thus, understanding the link between incentives and high school achievement remains a priority for research. To this end, my results show that banning affirmative action potentially reverses many of the gains minorities have attained in recent decades.

While scholarships reduce the monetary cost of attending cost, scholarships are only given to students who attend college. Thus we should not interpret scholarship programs as reducing the investment cost for minority students since programs that reduce the investment cost do so for all students regardless of college attendance.

[24]For Texas's Longhorn Scholars Program provides scholarships and additional academic counselling and planning services for qualified minority students, http://www.utexas.edu/student/connexus/scholars/index.html.

By explicitly modelling the investment incentives and decisions of college applicants, I analyze the effects of different admissions policies on applicants' secondary school achievement and human capital formation. The main results of this paper are that banning affirmative reduces aggregate human capital investment of the minority group, and when social welfare is defined over human capital investments, decreases the social welfare of the minority group. Additionally, I show that the black–white test score gap increases and give a specific prediction about how test score distributions change when affirmative action is prohibited, suggesting avenues for empirical research.

The persistence of the test score gap is relevant in light of the probable end of affirmative action programs within 25 years, as suggested by Justice O'Connor in the *Grutter* opinion. As long as the test score gap persists and America's colleges and universities continue to demand diverse student bodies, affirmative action policies will be used, and their effects debated. My work shows that banning affirmative action policies reduces campus diversity, and leads to sub–optimal admissions policies.

Many alternatives to affirmative action have been proposed and implemented, with varying degrees of success. The most notable are the Top X-Percent plans, which guarantee admissions to any high school student ranking in the top x percent of their graduating class. Evidence has shown these types of programs to have minimal success, at the cost of institutional independence (see Kain and OBrien (2001) or Bucks (2002)). My results show that other publicly funded programs such as targeted minority scholarship funds (Longhorn Opportunity Scholarships) and early outreach programs (University of California Early Outreach Academic Program) will increase minority college attendance, but may diminish minorities' human capital investments.

10 Appendix

Proof: *(Proposition 3.1)*

Obvious, in text.

Proof: *(Lemma 3.1)*

Differentiate the first order condition (9) with respect to θ and obtain:

$$\frac{\partial h^*}{\partial t} = \frac{-Ag'(h^* - t)}{\frac{\partial^2 c(h^*,\theta)}{\partial h^2} - Ag'(h^* - t)}. \tag{14}$$

The second order condition in footnote 9 implies the denominator is greater than zero, and the symmetry of the distribution $G(\cdot)$ implies that the numerator is greater (less) than zero if h^* is greater (less) than t. Thus, when $h^* > t$, $\frac{\partial h^*}{\partial t} > 0$ and when $h^* < t$, $\frac{\partial h^*}{\partial t} < 0$. If $h^* = t$, then the slope of the pdf $g(\cdot)$ is zero and $\frac{\partial h^*}{\partial t} = 0$.

Symmetry of the $G(\cdot)$ also implies that the denominator is larger than the numerator as h^* is greater than t, so when $h^* > t$, $\frac{\partial h^*}{\partial t} < 1$.

QED

To study college behavior, it is necessary to study first the shapes of the $M(\cdot)$ and $T(\cdot)$ functions. Lemmas 10.1 and 10.2 give the slope of the $M(\cdot)$ and $T(\cdot)$ functions.

Lemma 10.1 *The proportion of applicants admitted from either group decreases with threshold increases. That is,* $M_j'(\cdot) < 0$.

Proof: *(lemma 10.1)*

The proof is given for a generic $M(\cdot)$ function, and it applies equally to $M_W(\cdot)$ and $M_B(\cdot)$. Pointwise differentiation of (5) or (6) with respect to t gives:

$$M'(t) = \int_0^1 f(\theta)g(h^*(t,\theta) - t)(\frac{\partial h^*}{\partial t} - 1)d\theta, \tag{15}$$

By Lemma 3.1, $\frac{\partial h^*}{\partial t} - 1$ is less than zero for all θ. Thus the integrand of (16) is less than zero for all θ, establishing $M'(\cdot) < 0$.

QED

Lemma 10.2 *If* $\frac{\frac{\partial h_j^*}{\partial t_j}}{\frac{\partial h_j^*}{\partial t_j}-1} > -\frac{a(h_j^*,\theta)}{\frac{\partial a(h_j^*,\theta)}{\partial h}}\frac{g(h_j^*-t)}{G(h_j^*-t)}$ *for all* t *and* θ*, then the total quality of applicants admitted from either group decreases with threshold increases. That is,* $T_j'(\cdot) < 0$*.*

Proof: *(Lemma 10.2)*

The proof is given for a generic $T(\cdot)$ function, and it applies equally to $T_W(\cdot)$ and $T_B(\cdot)$. Pointwise differentiation of (3) (4) with respect to t gives:

$$T'(t) = \int_0^1 f(\theta)[\frac{\partial a}{\partial h}\frac{\partial h^*}{\partial t}G(h^*(t,\theta)-t) + a(h^*(t,\theta),\theta)g(h^*(t,\theta)-t)(\frac{\partial h^*}{\partial t}-1)]d\theta, \quad (16)$$

By Lemma 3.1, $\frac{\partial h^*}{\partial t}-1$ is less than zero for all θ and the second term in the brackets is negative. $\frac{\partial a}{\partial h} > 0$ by assumption, but for some t, $\frac{\partial h^*}{\partial t} > 0$, and the first term may be positive for some t. If $\frac{\frac{\partial h^*}{\partial t}}{\frac{\partial h^*}{\partial t}-1} > -\frac{a}{\frac{\partial a}{\partial h}}\frac{g}{G}$ for all t and θ then the integrand of (16) is less than zero for all t and θ, the result that $T'(\cdot) < 0$ follows.

QED

Proof: *(Proposition 4.1)*

Recall the definition of $\tilde{t}_W$ and $\tilde{t}_B$: $S = M_W(\tilde{t}_W) + M_B(\infty) = M_W(\tilde{t}_W)$ and $S = M_W(\infty) + M_B(\tilde{t}_B) = M_B(\tilde{t}_B)$. That is, $\tilde{t}_W$ and $\tilde{t}_B$ are the thresholds that satisfy the capacity constraint, such that the entire class is admitted from one group.[25]

For any S, the college may choose any pair thresholds which obey the capacity constraint, $S = M_W(t_W) + M_B(t_B)$. It may choose $t_W^* = \infty$ and $t_B^* = \tilde{t}_B$. Then, $Q(T_W(\infty), T_B(\tilde{t}_B)) = Q(0, T_B(\tilde{t}_B)) = 0$, so $(t_W^*, t_B^*) = (\infty, \tilde{t}_B)$ are not optimal because $Q(T_W(\tilde{t}_W), T_B(\tilde{t}_B)) > 0$ and $(\tilde{t}_W, \tilde{t}_B)$ are feasible. Similarly, $(t_W^*, t_B^*) = (\tilde{t}_W, \infty)$ are not optimal. Thus, there exists at least one maximum on the interior of the feasible set.

QED

Proof: *(Proposition 4.2)*

The proof is by contradiction. Recall the Lagrangian from the college's maximization problem, and the associated first order condition given in equation (12):

$$\frac{\frac{\partial Q(T_W,T_B)}{\partial T_B}T_B'}{\frac{\partial Q(T_W,T_B)}{\partial T_W}T_W'} = \frac{M_B'}{M_W'} \quad (17)$$

[25]Note that $M(\infty) = 0$. Also, Recall the assumption that $b > S$, that there are sufficient Bs to fill the entire class, so that $\tilde{t}_B$ is well defined.

Suppose $t_W^* \leq t_B^*$. Then it must be the case that $T_W > T_B$. Then, symmetry and concavity of the quality function imply that $\frac{\partial Q(T_B,T_B)}{\partial T_W} > \frac{\partial Q(T_W,T_B)}{\partial T_W}$. Combining the inequalities and rearranging, we have $\frac{\partial Q(T_W,T_B)}{\partial T_B} > \frac{\partial Q(T_W,T_B)}{\partial T_W}$. If the quality function is sufficiently concave then $\frac{\partial Q(T_W,T_B)}{\partial T_B} >> \frac{\partial Q(T_W,T_B)}{\partial T_W}$. Since $\frac{T_W'}{T_B'}\frac{M_B'}{M_W'}$ is bounded, this implies that first order condition in equation (17) is not satisfied. It must be the case that $t_W^* > t_B^*$.

QED

Proof: *(Proposition 5.1)*

The proof relies on revealed preference. The college may choose equal thresholds under affirmative action. If it does not choose equal thresholds, then it must have done so in order to increase college quality.

QED

Proof: *(Proposition 5.2)*

The proof is simple. Since $t_B^* < t_W^*$, then when affirmative action is banned the threshold increases for the minority and decreases for the majority. We know from lemma 10.1 that the mass of applicants admitted from each group is decreasing with the corresponding threshold. Thus the minority representation must decrease when affirmative action is banned.

QED

Proof: *(Lemma 6.1)*

Consider thresholds $\bar{t}$ and $\underline{t}$ as defined in section 6. To show that distributions cross exactly once for $(t_1, t_2) \in (\underline{t}, \bar{t}) \times (\underline{t}, \bar{t})$, we must show that $F(\theta(h, t_1)) = F(\theta(h, t_2))$ has a unique solution h for all (t_1, t_2). This is equivalent to showing that $\theta(h, t_1) = \theta(h, t_2)$ has a unique solution h for all (t_1, t_2), which is in turn equivalent to showing that $h^*(t_1, \theta) = h^*(t_2, \theta)$ has a unique solution θ for all (t_1, t_2).

Since $h^*(\cdot)$ is continuous, $h^*(t_1, \theta) = h^*(t_2, \theta)$ does have a solution for all θ. To show that the solution is unique, it sufficient to show that the cross partial derivative of $h^*(\cdot, \dot{})$ is negative. The cross partial can be found by appropriate differentiation of the first order condition from the applicant's maximization problem (drop the arguments to the functions):

$$\frac{\partial^2 h^*}{\partial t \partial \theta} = \frac{\frac{\partial h^*}{\partial \theta}[Ag''(h^*-t)(\frac{\partial h^*}{\partial t}-1) - \frac{\partial^3 c}{\partial h^3}\frac{\partial h^*}{\partial t}] - \frac{\partial^3 c}{\partial h^2 \partial \theta}\frac{\partial h^*}{\partial t}}{Ag'(h^*-t) - \frac{\partial^2 c}{\partial h^2}}. \tag{18}$$

If $\frac{\partial^3 c}{\partial h^3} = 0$ and $\frac{\partial^3 c}{\partial h^2 \partial \theta} = 0$, then the condition reduces to:

$$\frac{\partial^2 h^*}{\partial t \partial \theta} = \frac{\frac{\partial h^*}{\partial \theta} Ag''(h^*-t)(\frac{\partial h^*}{\partial t}-1)}{Ag'(h^*-t) - \frac{\partial^2 c}{\partial h^2}}. \tag{19}$$

Since the denominator is less than zero (second order condition), $\frac{\partial h^*}{\partial \theta} > 0$ and $\frac{\partial h^*}{\partial t} - 1 < 0$ (assumption), the sign of $\frac{\partial^2 h^*}{\partial t \partial \theta}$ (lemma 3.1) is equal to the sign of $g''(h^* - t)$. If $g''(h^*(\overline{t}, \underline{\theta}) - \overline{t}) < 0$ and $g''(h^*(\underline{t}, \overline{\theta}) - \underline{t}) < 0$, then $g''(h^* - t) < 0$ for all θ and t. Thus, $\frac{\partial^2 h^*}{\partial t \partial \theta} < 0$ establishing that investment distributions cross exactly once for $(t_1, t_2) \in (\underline{t}, \overline{t}) \times (\underline{t}, \overline{t})$.

QED

Before giving the proof of proposition 6.1, we need one additional lemma. When distributions cross exactly once, we may modify the definition of second order dominance:

Lemma 10.3 *If cumulative distributions $F(h)$ and $G(h)$ cross exactly once, then $F(h)$ second order dominates $G(h)$ if and only if $\int_0^{\overline{h}} F(h)dh < \int_0^{\overline{h}} G(h)dh$.*

Proof: *(Lemma 10.3)*

Clearly, if $F(h)$ is dominant over $G(h)$, then $\int_0^{\overline{h}} F(h)dh < \int_0^{\overline{h}} G(h)dh$ holds. The proof in the other direction is as follows. Let the two distributions $G(h)$ and $F(h)$ cross at $h = \tilde{h}$, and suppose $t_2 > t_1$. Then we know that for all $h < \tilde{h}$, $G(h) > F(h)$ (since types choosing $h < \tilde{h}$ decrease their investments when the threshold increases). Therefore, $\int_0^{\tilde{h}}[G(h) - F(h)]de$ is greater than zero. Similarly, for all $h > \tilde{h}$, $G(h) < F(h)$. Therefore, for $h' > \tilde{h}$, $\int_{\tilde{h}}^{h'}[G(h) - F(h)]dh$ is less than zero and decreasing with h' (equal to zero if $h' = \tilde{h}$). We have shown that for $h' > \tilde{h}$, $\int_0^{h'}[G(h) - F(h)]dh$ is decreasing in h'. Thus if $\int_0^{\overline{h}}[F(h) - G(h)]dh$ is greater than zero, then $\int_0^{\hat{h}}[G(h) - F(h)]dh$ is greater than zero for all $\hat{h}$, satisfying the definition of second order dominance of $F(h)$ over $G(h)$.

QED

Proof: *(Proposition 6.1, part iii.)*

Using lemma 10.3, we only need to check the SOSD condition at the upper bound of the support of h. Graphically, this condition is shown in figure 10 and amounts to checking that area C is greater than area D.

Consider the threshold $\bar{t}$ and the investment distribution it induces. Consider a marginal decrease in the threshold to $t_2 < \bar{t}$. This change in threshold will cause a very small number of applicants to decrease their investments (very high ability applicants), and a large number of applicants to increase their investments (low and middle ability applicants). Additionally, the downward change in investments of the high ability applicants will be small compared to the upward change in investments of the low ability applicants, since the slope of $h^*(t, \overline{\theta})$ at $t = \bar{t}$ is zero and the slope of $h^*(t, \underline{\theta})$ at $t = \bar{t}$ is strictly less than zero. Thus, it must be the case that $F(h|t_2)$ SOSD $F(h|\bar{t})$. Continue the process inductively until SOSD is not satisfied to find $\hat{t}$ (Also note that the SOSD relation is transitive, i.e. if $F()$ SOSD $G()$, and $G()$ SOSD $H()$, then $F()$ SOSD $H()$).

QED

Proof: *(Proposition 6.2)*

We know from proposition 4.2 that when affirmative action is banned, selective colleges raise the admissions threshold for the majority and lower the threshold for the minority. For sufficiently selective colleges, proposition 6.1 gives the desired result.

QED

Proof: *(Proposition 6.3)*

Proposition 6.1 tells us that for sufficiently selective colleges we may rank thresholds in terms of second order stochastic dominance, and that the lower threshold will induce the dominant distribution when comparing any two thresholds. Proposition 4.2 tells us that affirmative action is characterized by a higher threshold for the majority and a lower threshold for the minority (when compared to the no-affirmative action case). Thus affirmative action induces a dominant distribution for the minority and a dominated distribution for the majority (when compared to the no-affirmative action case). To obtain the result, we cite a known result from Foster and Shorrocks (1988) which states that $\int u(x)dF(x) > \int u(x)dG(x)$ if and only if $F(\cdot)$ SOSD $G(\cdot)$, for $u(\cdot)$ increasing and strictly concave.

QED

Proof: *(Proposition 7.1)*

Let $E(s|t)$ be the expected test score for a given threshold. Since $s = h + \eta$, where h and η are uncorrelated and η is mean zero, then it is obvious that $E(h|t) = E(s|t)$. Then we simply need to apply proposition 6.2 to obtain the result.

QED

Before beginning the proof of proposition 7.2, recall that investment distributions may cross each other once at most (see lemma 6.1), and the modified definition of second order dominance when distributions cross once (see lemma 10.3).

Proof: *(Proposition 7.2)*

The proof will be stated in terms of the random variables h, h_1, η, η_1, $s = h + \eta$ and $s_1 = h_1 + \eta_1$. Let the distribution of h be $F(\cdot)$ with support $[0, 1]$, the distribution of h_1 be $\hat{F}(\cdot)$ with support $[0, 1]$, and the distribution of η and η_1 be $G(\cdot)$ with support $[-\infty, \infty]$.

Suppose that h SOSD h_1. Then it must be that for all $\hat{h} \in [0, 1]$:

$$\int_0^{\hat{h}} F(h)dh \leq \int_0^{\hat{h}} \hat{F}(h)dh \tag{20}$$

Using the convolution formula, the distribution of s is $H(s) = \int_{-\infty}^{\infty} g(\eta)F(s - \eta)d\eta$, and the distribution of s_1 is $\hat{H}(s_1) = \int_{-\infty}^{\infty} g(\eta)\hat{F}(s_1 - \eta)d\eta$. Thus, for s to SOSD s_1, the following must be true. For all $\hat{s} \in [-\infty, \infty]$:

$$\int_{-\infty}^{\hat{s}} \int_{-\infty}^{\infty} g(\eta)F(s - \eta)d\eta ds \leq \int_{-\infty}^{\hat{s}} \int_{-\infty}^{\infty} g(\eta)\hat{F}(s - \eta)d\eta ds, \tag{21}$$

which can be written as:

$$\begin{aligned}
\int_{-\infty}^{\hat{s}} \int_{-\infty}^{\infty} g(\eta)[F(s - \eta) - \hat{F}(s - \eta)]d\eta ds &\leq 0, &(22)\\
\int_{-\infty}^{\infty} \int_{-\infty}^{\hat{s}} g(\eta)[F(s - \eta) - \hat{F}(s - \eta)]ds d\eta &\leq 0, &(23)\\
\int_{-\infty}^{\infty} g(\eta)\Big[\int_{-\infty}^{\hat{s}} F(s - \eta) - \hat{F}(s - \eta)ds\Big]d\eta &\leq 0. &(24)
\end{aligned}$$

Since h SOSD h_1, we know that equation (20) holds, which implies that the bracketed term in (24) is less than zero for all $\hat{s}$. Then, since $g(\cdot)$ is positive for all values in its support, the inequality in equation (24) must hold for all $\hat{s}$.

Now suppose that s SOSD s_1. I will show that SOSD of s over s_1 implies that the inequality in equation (20) holds, that h SOSD h_1. Using the convolution formula, we can write the distribution of s as $H(s) = \int_0^1 f(h)G(s-h)dh$, and the distribution of s_1 as $\hat{H}(s_1) = \int_0^1 \hat{f}(h)G(s_1 - h)dh$. Then SOSD implies:

$$\int_{-\infty}^{\hat{s}} \int_0^1 f(h)G(s-h)dhds \leq \int_{-\infty}^{\hat{s}} \int_0^1 \hat{f}(h)G(s-h)dhds, \tag{25}$$

which can be written as:

$$\begin{aligned}
\int_{-\infty}^{\hat{s}} \int_0^1 f(h)G(s-h)dhds - \int_{-\infty}^{\hat{s}} \int_0^1 \hat{f}(h)G(s-h)dhds &\leq 0, &(26)\\
\int_0^1 \int_{-\infty}^{\hat{s}} f(h)G(s-h)dsdh - \int_0^1 \int_{-\infty}^{\hat{s}} \hat{f}(h)G(s-h)dsdh &\leq 0, &(27)\\
\int_0^1 f(h)\Big[\int_{-\infty}^{\hat{s}} G(s-h)ds\Big]dh - \int_0^1 \hat{f}(h)\Big[\int_{-\infty}^{\hat{s}} G(s-h)ds\Big]dh &\leq 0. &(28)
\end{aligned}$$

Let $v = F(\cdot)$, so $dv = f(\cdot)$ and $u = \int_{-\infty}^{\hat{s}} G(s-h)ds$ so $du = -\int_{-\infty}^{\hat{s}} g(s-h)ds$, and integrate each term in equation (28) by parts to obtain:

$$\begin{aligned}
&\Big[F(1)\int_{-\infty}^{\hat{s}} G(s-h)ds - \int_0^1 F(h)\Big[-\int_{-\infty}^{\hat{s}} g(s-h)ds\Big]dh\Big]\\
&\qquad -\Big[\hat{F}(1)\int_{-\infty}^{\hat{s}} G(s-h)ds - \int_0^1 \hat{F}(h)\Big[-\int_{-\infty}^{\hat{s}} g(s-h)ds\Big]dh\Big] \leq 0, \qquad (29)
\end{aligned}$$

and rearrange (note that $F(1) = 1$, and $\hat{F}(1) = 1$):

$$\begin{aligned}
\int_0^1 F(h)\Big[\int_{-\infty}^{\hat{s}} g(s-h)ds\Big]dh - \int_0^1 \hat{F}(h)\Big[\int_{-\infty}^{\hat{s}} g(s-h)ds\Big]dh &\leq 0, &(30)\\
\int_0^1 [F(h) - \hat{F}(h)]\Big[G(\hat{s}-h) - G(-\infty-h)\Big]dh &\leq 0, &(31)\\
\int_0^1 [F(h) - \hat{F}(h)]G(\hat{s}-h)dh &\leq 0. &(32)
\end{aligned}$$

Since s SOSD s_1, equation (32) must be true for all $\hat{s}$. So, let $\hat{s} = \infty$. Then, $G(\infty - h) = 1$ for all h, and it must be true that:

$$\int_0^1 [F(h) - \hat{F}(h)]dh \leq 0. \tag{33}$$

Now, recall lemma 10.3 which stated that if distributions cross exactly once, then in order to show second order stochastic dominance it is sufficient to check that the inequality given in equation (20) holds for $\hat{h} = 1$ (when distributions do not cross, the same test is clearly sufficient). Equation (33) shows exactly that.

QED

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