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Savings and Economies of Marriage: Intra-Marriage Financial Distributions as Determinants of Savings*

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This paper presents an inter-temporal model of individual savings behavior with intra-marriage financial distributions and uncertainty about marriage and divorce. Spouses paying these distributions experience diseconomies of marriage; the spouses who receive them benefit from economies of marriage. We show that higher likelihoods of marriage and divorce are associated with lower (higher) personal savings rates in the presence of diseconomies (economies) of marriage. These results hint at the possibility that structural patterns of social behavior may be at the root of the lack of effectiveness of conventional policy instruments, such as tax incentives, to promote personal savings.

JEL Codes: E21, J12
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1. Introduction

Personal savings behavior is a very important variable influencing economic growth. Conventional economic theory, however, does not seem to provide a good understanding of its determinants. For instance, there are currently no good explanations for the considerable decrease in personal savings rates observed in the United States over the last few decades or for the large differences in personal savings observed among industrialized countries [see, for example, Auerbach and Kotlikoff (1990) and Guidolin and La Jeunesse (2007)]. Even more importantly, there is growing concern about the lack of effectiveness of conventional policy instruments, such as tax incentives, when it comes to promoting personal savings [see, for example, Tanzi and Zee (1999) and Hungerford (2006)].

Some of the most commonly used explanations for variations in personal savings are related to demographic characteristics. Using a life-cycle model of consumption and savings, economists have previously related personal savings to, for example, the age structure of the population, life expectancy, labor force participation rate of the aged, retirement age, family size, and percent of women in the labor force [see, for example, Apps and Rees (2010), Horioka (1989, 1997), Horioka and Terada-Hagiwara (2010), Horioka and Wan (2007), and Smith (1990)]. In addition, a number of empirical studies have established an association between marriage and personal savings [see, for example, Auerbach and Kotlikoff (1990), Lupton and Smith (2003), Kureishi and Wakabayashi (2013), and Zissimopoulos, Karney and Rauer (2013)].

We present an inter-temporal model of personal savings behavior that postulates the existence of intra-marriage financial distributions leading some spouses to have personal unobservable disposable income beyond their personal observed income. We call this economies of marriage. Such economies allow people to save a higher fraction of their personal income have a flip side for the spouses who 'pay' the distributions and therefore experience diseconomies of marriage. With a lower unobservable disposable personal income the latter will have a lower personal propensity to save out of personal observed income.

We follow the tradition of the *New Home Economics* pioneered by Becker (1960, 1973) and Mincer (1962, 1963) in modeling marriages as small non-profit firms. In addition, we follow the Demand and Supply model in Becker's (1973) theory of marriage analyzing the determinants of individual access to the gain from marriage. Access to the gain from marriage typically includes intra-marriage financial distributions. He shows how such intra-marriage distributions are a function of many of the same factors that explain wages and other distributions in commercial firms. To the extent that we view intra-marriage distributions as compensations in exchange for individual contributions to household production our paper is related to the work of Grossbard-Shechtman (1984), Apps and Rees (1997) and Chiappori (1997).

2. The Analytical Framework

Our stylized agent lives for three periods: youth/period 1, midlife/period 2, and old age/period 3. The agent maximizes an individual inter-temporal utility function (1) defined on consumption c_i with i = 1, 2, and 3. The utility function satisfies positive and decreasing marginal utility, and constant absolute risk aversion. Future consumption is discounted by a factor δ that captures impatience.

(1) $$Max[U(c_1) + \delta U(c_2) + \delta^2 U(c_3)]$$

The agent can derive income from two sources: work or interest on past savings. It is assumed that the agent earns the same wage w in every one of the three periods, which assumes inter-temporal smoothing of earnings. Savings earn a return r. In the absence of intergenerational altruism and/or uncertainty as to the time of death, there are no savings in period 3.

Various marital states are possible. We assume that the agent is unmarried in the first period. In periods 2 and 3 (midlife and old age), the budget constraint depends on the agent's marital status. There is a probability p_2 that the agent will remain single in the second period, the alternative being to marry. There is a probability p_3 that the agent will be single, widowed, or divorced in the third period. For the sake of simplicity we will refer to p_3 as the probability of divorce. It is assumed that p_2 and p_3 are exogenous and independent, that the divorced state is equivalent to the single state, that agents can not marry in period 3, and that divorce does not entail either extra expenses or extra income.

Consumption and savings are personal, not household based. How much the agents can buy based on their earnings is captured by the 'economies of marriage' parameter, γ. If $\gamma > 1$ there are economies of marriage in that individual income has a greater purchasing power for agents if they are married than if they are single, as they may have access to their spouse's income or benefit by consuming goods that the spouse purchased. In contrast, if $\gamma < 1$, there are diseconomies of marriage in that individual income has a lower purchasing power if the agent is married, as agent's income is used to 'pay' the spouse or the spouse buys goods that don't benefit the agent.

Given these considerations, the budget constraints for the three periods can be written as:

(2) $$w = c_1 + s_1$$

$w+(1+r)s_1 = c_2 + s_2$ with probability p_2

$\gamma w+(1+r)s_1 = c_2 + s_2$ with probability (1- p_2)

$w+(1+r)s_2 = c_3$ with probability p_3

$\gamma w+(1+r)s_2 = c_3$ with probability (1- p_3)

The optimal level of savings in the first two periods (s_1, s_2) is obtained by maximizing (1) subject to (2) and can be presented in general terms as the ArgMax of V:

$$V = U(w-s_1) + \delta\{p_2 U[w+(1+r)s_1 - s_2] + (1-p_2)U[\gamma w + (1+r)s_1 - s_2]\}$$

(3)

$$+\delta^2\{p_3 U[w+(1+r)s_2] + (1-p_3\}U[\gamma w+(1+r)s_2]\}$$

Given the properties of the utility function (s_1, s_2) are determined by

(4) $$\frac{\partial V}{\partial s_1} = -U'(w-s_1) + \delta(1+r)\{p_2 U'[w+(1+r)s_1 - s_2] + (1-p_2)U'[\gamma w+(1+r)s_1 - s_2]\} = 0$$

(5) $$\frac{\partial V}{\partial s_2} = -\delta\{p_2 U'[w+(1+r)s_1 - s_2] + (1-p_2)U'[\gamma w+(1+r)s_1 - s_2]\}$$

$$+\delta^2(1+r)\{p_3 U'[w+(1+r)s_2] + (1-p_3)U'[\gamma w+(1+r)s_2]\} = 0$$

If these conditions are denoted in an implicit form by $F_1(s_1, s_2; p_2, p_3) = 0$ and $F_2(s_1, s_2; p_2, p_3) = 0$, the effects of the probabilities of marriage and divorce on savings behavior can be obtained by using the implicit function theorem and the related information presented in the Appendix.

3. On the Effects on Savings of Changes in the Probability of Marriage and of Divorce

Proposition 1 [Effects of changes in the probability of marriage on savings at youth]

Savings at youth decrease with the probability of marriage for agents anticipating economies of marriage ($\gamma > 1$) *and increase with the probability of marriage for agents anticipating diseconomies of marriage* ($\gamma < 1$).

Proof: Using standard techniques we obtain

(6) $$\frac{\partial s_1}{\partial p_2} = \left(-\frac{\partial F_1}{\partial p_2} \cdot \frac{\partial F_2}{\partial s_2} + \frac{\partial F_2}{\partial p_2} \cdot \frac{\partial F_1}{\partial s_2}\right) / \det(F_s)$$

It can be shown that

$$sign\left(\frac{\partial s_1}{\partial p_2}\right) = sign\left\{\frac{\partial F_1}{\partial p_2} + \frac{\partial F_2}{\partial p_2}\left[\frac{\partial F_1}{\partial s_2} / \left(-\frac{\partial F_2}{\partial s_2}\right)\right]\right\}$$

where the term in the square bracket is positive. Furthermore, the reason why the sign is ambiguous is because $sign\frac{\partial F_1}{\partial p_2} \neq sign\frac{\partial F_2}{\partial p_2}$ regardless of the value of γ.

Now it can be shown that:

$$sign\left(\frac{\partial s_1}{\partial p_2}\right) = sign\left\{(1+r) - \left[\frac{\partial F_1}{\partial s_2} / \left(-\frac{\partial F_2}{\partial s_2}\right)\right]\right\} \; if \;\; \gamma > 1 \, and$$

$$sign(\frac{\partial s_1}{\partial p_2}) \neq sign\{(1+r)-[\frac{\partial F_1}{\partial s_2}/(-\frac{\partial F_2}{\partial s_2})]\}\ if\ \gamma < 1$$

From the derivatives above and (A.2') it can be seen that $[\frac{\partial F_1}{\partial s_2}/(-\frac{\partial F_2}{\partial s_2})] = (1+r)B$, where $0 < B < 1$ (numerator is lower in absolute value than the denominator).

Accordingly,

$$\frac{\partial s_1}{\partial p_2} > 0\ if\ \gamma > 1$$

$$\frac{\partial s_1}{\partial p_2} < 0\ if\ \gamma < 1.$$ **QED**

The intuition behind this proposition is that agents expecting economies of marriage save less in youth, when they anticipate a state of the world with a higher purchasing power . The reverse is true for agents expecting diseconomies of marriage.

Proposition 2 [Effects of changes in the probability of marriage on savings in midlife]

Savings in midlife increase with the probability of marriage for agents experiencing economies of marriage ($\gamma > 1$) *and decrease with the probability of marriage for agents expecting diseconomies of marriage* ($\gamma > 1$) .

Proof:

(7) $$\frac{\partial s_2}{\partial p_2} = (-\frac{\partial F_1}{\partial s_1}\cdot\frac{\partial F_2}{\partial p_2}+\frac{\partial F_1}{\partial p_2}\cdot\frac{\partial F_2}{\partial s_1})/\det(F_s)$$

It can be shown that

$$sign(\frac{\partial s_2}{\partial p_2}) = sign\{\frac{\partial F_1}{\partial p_2}+\frac{\partial F_2}{\partial p_2}[(-\frac{\partial F_1}{\partial s_1})/\frac{\partial F_2}{\partial s_1})]\}$$

where the term in the square bracket is positive. Now, it can also be shown that

$$sign(\frac{\partial s_2}{\partial p_2}) = sign\{(1+r)-[(-\frac{\partial F_1}{\partial s_1})/\frac{\partial F_2}{\partial s_1})]\}\ if\ \gamma > 1$$

$$sign(\frac{\partial s_2}{\partial p_2}) \neq sign\{(1+r)-[(-\frac{\partial F_1}{\partial s_1})/\frac{\partial F_2}{\partial s_1}]\}\ if\ \gamma < 1.$$

From the derivatives above and (A.6) it can be seen that $[(-\frac{\partial F_1}{\partial s_1})/\frac{\partial F_2}{\partial s_1}] = (1+r)+C$, where $C > 0$ (numerator is lower in absolute value than the denominator). Accordingly,

$$\frac{\partial s_2}{\partial p_2} < 0\ if\ \gamma > 1$$

$$\frac{\partial s_2}{\partial p_2} > 0\ if\ \gamma < 1.$$ **QED**

The impact of a change in the probability of marriage on individual savings in midlife is the opposite of its impact on savings in youth. An agent who expects diseconomies of marriage will save more in youth, before marriage. However, once married and actually experiencing these diseconomies, he or she will save less. The opposite is true with economies of marriage.

Proposition 3 [Effects of changes in the probability of marriage on lifetime savings]

Consider an agent with a relatively high rate of impatience (δ sufficiently lower than $1/(1+r)$). Then, an increase in the probability of marriage increases the present value of lifetime savings for agents expecting economies of marriage and decreases that present value if diseconomies of marriage are expected [unless both the probability of divorce and economies of marriage are very low].

Proof:

From (6), (7), (A.3), and (A.8)

$$sign[\frac{\partial s_1}{\partial p_2}+\frac{1}{1+r}\cdot\frac{\partial s_2}{\partial p_2}] = sign[\frac{\partial F_1}{\partial s_1} - (1+\mathrm{r})^2\frac{\partial F_2}{\partial s_2}]\, if\ \gamma>1, (\neq if\ \gamma<1)$$

In turn, using (A.1) and (A.7)

$$sign[\frac{\partial F_1}{\partial s_1} - (1+\mathrm{r})^2\frac{\partial F_2}{\partial s_2}] =$$

$$sign[U''(w-s_1)-\delta^2(1+r)^4\{p_3U''[w+(1+r)s_2]+(1-p_3)U''[\gamma w+(1+r)s_2]\}]$$

Suppose now that the rate of time impatience is such that $\delta^2(1+r)^4=1$ and as assumed that $U(\cdot)$ displays constant relative risk aversion c, such that $\frac{U''(\cdot)}{U'}=-c$.

In this case

$$sign[\frac{\partial F_1}{\partial s_1} - (1+\mathrm{r})^2\frac{\partial F_2}{\partial s_2}] =$$

$$sign[p_3U'[w+(1+r)s_2]+(1-p_3)U'[\gamma w+(1+r)s_2]-U'(w-s_1)]$$

Notice that $w-s_1<w+(1+r)s_2$, and from concavity of $U(\cdot)$, for a sufficiently high p_3, the sign of the above expression is negative. Moreover, if $\gamma>1$, then the sign of the above equation is unambiguously negative since the first two terms are a convex combination of two terms each of which is lower than the third term in absolute value. In this case the magnitude of the probability of divorce does not matter. On the other hand, if $\gamma<1$, the sign of the equation will also be negative unless both γ and p_3 are very low.

It should be noted that in reality one would expect $\delta^2(1+r)^4>1$. For this proposition to hold it is sufficient that the discount rate is relatively low (a rate of time impatience relatively high) such that this expression is not much larger than one. **QED**

For a relatively impatient agent the present value of lifetime savings increases with the probability of marriage in the presence of economies of marriage and decreases in the presence of diseconomies of marriage. Overall, individuals experiencing economies of marriage have higher lifetime purchasing power, despite the fact that they save less when they are young. Intuitively, a high degree of impatience guarantees that savings are reduced less at youth than are increased at maturity. The opposite is true for agents with diseconomies of marriage. An interesting exception, however, is that even for such agents if the probability of divorce and diseconomies of marriage are very low, an increase in the probability of marriage will also increase lifetime savings. Intuitively, the diseconomies of marriage are dramatic and the likelihood of returning to a higher purchasing power state is small so anticipated future purchasing power from marriage is relatively low and greater savings are required.

Proposition 4 [Effect of the probability of divorce on savings]
Savings of agents throughout their lifetimes decrease with the probability of divorce if they experience or anticipate diseconomies of marriage ($\gamma < 1$). *In the presence of economies of marriage* ($\gamma > 1$) *a higher probability of divorce affects savings positively. Furthermore, the probability of divorce affects savings behavior (negatively or positively) the most at midlife.*
Proof:

(8) $$\frac{\partial s_1}{\partial p_3} = (\frac{\partial F_1}{\partial s_2} \cdot \frac{\partial F_2}{\partial p_3}) / \det(F_s)$$

(9) $$\frac{\partial s_2}{\partial p_3} = (-\frac{\partial F_1}{\partial s_1} \cdot \frac{\partial F_2}{\partial p_3}) / \det(F_s)$$

Notice that given the results above,

$$sign(\frac{\partial s_1}{\partial p_3}) = sign(\frac{\partial s_2}{\partial p_3}) = sign(\frac{\partial F_2}{\partial p_3})$$

which is positive if $\gamma > 1$ and negative if $\gamma < 1$.

Notice also that from (A.1) and (A.2), $-\frac{\partial F_1}{\partial s_1} > \frac{\partial F_1}{\partial s_2}$, which implies that $\frac{\partial s_1}{\partial p_3} < \frac{\partial s_2}{\partial p_3}$, i.e., the probability of divorce affects savings behavior (negatively or positively) more at midlife than in youth. **QED**

The intuition behind this proposition is that agents who experience economies of marriage view divorce as representing lower income. Therefore, the higher the likelihood of divorce, the more they have to save to hedge against lower income in the future.

4. Conclusions

An inter-temporal model of personal savings with uncertainty about marriage and divorce was presented in which agents make individual decisions to save out of their disposable individual incomes. The size of that income depends on whether they anticipate receiving distributions from their spouse or paying distributions to the spouse. In the former case their disposable personal income will exceed their own observed income, resulting in economies of marriage. If they expect to have to pay distributions to their spouse their disposable income is likely to be lower than their observed income, i.e. they will experience diseconomies of marriage. We show that higher probabilities of marriage and divorce are associated with lower (greater) personal savings rates in the presence of diseconomies (economies) of marriage. It follows that the likelihood of marriage and divorce and the expectation of economies or diseconomies of marriage are determinants of individual savings behavior, the predicted effect of marriage or divorce being conditional on whether agents expect to receive or pay intra-marriage financial distributions.

Who pays and who receives intra-marriage distributions and the size of such distributions—the source of economies or diseconomies of marriage--has been overlooked in previous analyses of savings. It is worthy of further attention in studies of individual savings, as it helps explain savings differentials by gender and marital status. In traditional societies women are expected to perform more household production in marriage than men and are therefore likely to experience economies of marriage. Therefore, our analysis helps explain why in Japan, a traditional society where women can expect economies of marriage, single women with a higher probability of marriage have been found to save less than their counterparts with a lower probability of marriage [see, for example, Kureishi and Wakabayashi (2013)].

Our model may also throw some light on the determinants of the aggregate personal savings behavior in an economy. For instance, consider an economy where the primary earners are also the primary savers. The more they share their purchasing power with their spouses—i.e. the higher their diseconomies of marriage--the more increases in the probability of divorce will translate into lower savings for the primary earner/saver. To the extent that parallel increases in savings rates among the spouses of these primary earners are smaller in absolute value than the decrease in primary earners' savings, the aggregate personal savings rate is likely to drop when divorce rates increase. Accordingly, our results hint at the possibility that structural patterns of social behavior may be at the root of the lack of effectiveness of conventional policy instruments, such as tax incentives, to promote personal savings.

REFERENCES

Apps, Patricia and Ray Rees (1997), "Collective Labor Supply and Household Production," *Journal of Political Economy* 105(1): 178-90.

Apps, Patricia and Ray Rees (2010), "Family Labour Supply, Taxation and Savings in an Imperfect Capital Market," *Review of Economics of the Household* 8 (3): 297-323.

Auerbach, Alan J. and Lawrence J. Kotlikoff (1990), "Demographics, Fiscal Policy, and U.S. Savings in the 1980s and Beyond," in Lawrence H. Summers (Ed.). *Tax Policy and the Economy* Vol. 4, NBER, Cambridge: MIT Press.

Becker, Gary S. (1960), "An Economic Analysis of Fertility." In *Demographic and Economic Change in Developed Countries*, National Bureau Committee for Economic Research. Princeton University Press.

_____. (1973), "A Theory of Marriage: Part I." *Journal of Political Economy* 81:813-846.

Chiappori, Pierre-Andre, (1997). "Introducing Household Production in Collective Models of Labor Supply," *Journal of Political Economy* 105(1): 191-209.

Grossbard, Shoshana A. (1984), "A Theory of Allocation of Time in Markets for Labor and Marriage," *Economic Journal*, 94(4): 863-882.

Guidolin, Massimo and Elizabeth A. La Jeunesse (2007), "The Decline in the US Personal Savings Rate: Is It Real and Is It a Puzzle?" *Federal Reserve Bank of St. Louis Review* 89 (6):491-514.

Horioka, Charles Yuji (1989), "Why Is Japan's Private Saving Rate So High?" in R. Sato and T. Negishi, eds., *Developments in Japanese Economics* (Tokyo: Harcourt Brace Jovanovich), 145-178.

_____. (1997), "A Cointegration Analysis of the Impact of the Age Structure of the Population on the Household Saving Rate in Japan," *Review of Economics and Statistics* 79 (3):511-516.

_____ and Terada-Hagiwara, Akiko (2010), "Sources and Prospects for Asia's Growth: Saving Rate," mimeo., Asian Development Bank, Manila, Philippines.

_____ and Wan, Junmin (2007), "The Determinants of Household Saving in China: A Dynamic Panel Analysis of Provincial Data," *Journal of Money, Credit and Banking* 39 (8):2077-2096.

Hungerford, T. (2006), "Savings Incentives: What May Work, What May Not." Congressional Research Service Report. http://www.policyarchive.org/handle/10207/bitstreams/2867.pdf

Kureishi, Wataru and Midori Wakabayashi (2013) "What motivates single women to save? The case of Japan," *Review of Economics of the Household*, http://link.springer.com/article/10.1007/s11150-013-9191-z

Lupton, Joseph P. and James P. Smith (2003), "Marriage, Assets, and Savings." In Shoshana Grossbard-Shechtman (ed.) *Marriage and the Economy*. New York: Cambridge University Press.

Mincer, Jacob (1962), "Labor Force Participation of Married Women: a Study of Labor Supply." In *Aspects of Labor Economics* edited by H. Gregg Lewis. Princeton, N.J.: Princeton University Press.

_____ (1963), "Market Prices, Opportunity Costs, and Income Effects." In *Measurement in Economics* edited by C. Christ. Stanford, CA: Stanford University Press.

Smith, Roger S. (1990), "Factors Affecting Savings, Policy Tools, and Tax Reform: A Review." *International Monetary Fund Staff Paper* 37(1):1-70.

Tanzi, Vito and Howell Zee (1999), "Taxation and the Household Savings Rate: Evidence from OECD Countries," International Monetary Fund Working Paper 98/36.

Zissimopoulos, Julie, Benjamin Karney and Amy Rauer (2013) "Marriage and economic well-being at older ages," *Review of Economics of the Household*, http://link.springer.com/article/10.1007/s11150-013-9205-x

APPENDIX

The determinant of the Jacobian matrix of the first order conditions with respect to s_1 and s_2 is positive, i.e., $Det(F_s) > 0$. This is a direct requirement of the optimization problem in that it relates to the strict concavity of the objective function with respect to the decision variables and satisfies the conditions of the implicit function theorem. To obtain the necessary information for the identification of the effects on savings behavior we totally differentiate F_1 and F_2 to obtain:

(A.1) $$\frac{\partial F_1}{\partial s_1} = U''(w-s_1) + \delta(1+r)^2\{p_2U''[w+(1+r)s_1-s_2] + (1-p_2)U''[\gamma w+(1+r)s_1-s_2]\} < 0$$

(A.2) $$\frac{\partial F_1}{\partial s_2} = -\delta(1+r)\{p_2U''[w+(1+r)s_1-s_2] + (1-p_2)U''[\gamma w+(1+r)s_1-s_2]\} > 0$$

or using (A.1)

(A.2') $$-\frac{\partial F_1}{\partial s_1} = -U''(w-s_1) + (1+r)\frac{\partial F_1}{\partial s_2} > \frac{\partial F_1}{\partial s_2}$$

(A.3) $$\frac{\partial F_1}{\partial p_2} = \delta(1+r)\{U'[w+(1+r)s_1-s_2] - U'[\gamma w+(1+r)s_1-s_2]\} > 0 \; if \; \gamma > 1 \;\; (< 0 \; if \; \gamma < 1)$$

(A.4) $$\frac{\partial F_1}{\partial p_3} = 0$$

(A.5) $$\frac{\partial F_2}{\partial s_1} = \frac{\partial F_1}{\partial s_2} = -\delta(1+r)\{p_2U''[w+(1+r)s_1-s_2] + (1-p_2)U''[\gamma w+(1+r)s_1-s_2]\} > 0$$

(A.6) $$\begin{aligned}\frac{\partial F_2}{\partial s_2} &= \delta\{p_2U''[w+(1+r)s_1-s_2] + (1-p_2)U''[\gamma w+(1+r)s_1-s_2]\} \\ &\quad + \delta^2(1+r)^2\{p_3U''[w+(1+r)s_2] + (1-p_3)U''[\gamma w+(1+r)s_2]\} \\ &= -\frac{1}{1+r}\cdot\frac{\partial F_1}{\partial s_2} + \delta^2(1+r)^2\{p_3U''[w+(1+r)s_2] + (1-p_3)U''[\gamma w+(1+r)s_2]\} < 0\end{aligned}$$

or using (A.1), (A.2), and (A.6)

(A.6') $\frac{\partial F_2}{\partial s_2} = -\frac{1}{(1+r)^2} \cdot [U''(w-s_1) - \frac{\partial F_1}{\partial s_1}]$

$$+\delta^2 (1+r)^2 \{ p_3 U''[w+(1+r)s_2] + (1-p_3)U''[\gamma w + (1+r)s_2]\} < 0,$$

where given (A.1) the first term has a negative sign.

(A.7) $\frac{\partial F_2}{\partial p_2} = -\delta\{U'[w+(1+r)s_1 - s_2] - U'[\gamma w + (1+r)s_1 - s_2]\} < 0 \; if \; \gamma > 1 \; (> 0 \; if \; \gamma < 1)$

From (A.3) and (A.7) it follows that $\frac{\partial F_1}{\partial p_2} = -(1+r)\frac{\partial F_2}{\partial p_2}$ and these two derivatives will always have the opposite sign regardless of γ.

(A.8) $\frac{\partial F_2}{\partial p_3} = \delta^2\{U'[w+(1+r)s_2] - U'[\gamma w + (1+r)s_2]\} > 0 \; if \; \gamma > 1 \; (< 0 \; if \; \gamma < 1)$